GUY CARPENTER

SPECTRAL RISK MEASURES (SRM) AND APPLICATIONS IN INSURANCE ERM – PART 1

ASTIN WEBINAR

MARCH 25, 2020

John A. Major, ASA, MAAA
Director of Actuarial Research

New York

MARSH & McLENNAN COMPANIES

About this work…

- We are **NOT**
  - Addressing theoreticians
  - Explaining market prices
    - "What is the correct risk premium?"

About this work…

- We are **NOT**
  - Addressing theoreticians
  - Explaining market prices
    - "What is the correct risk premium?"

- We **ARE**
  - Addressing working actuaries
  - Presenting a framework for portfolio management
    - "Which business segments are over- or under-priced?"

Assumptions / strategy

- External rule determines assets required
- Law-invariant rule prices the portfolio
- Business segments get equal priority in default
- Ignore expenses, investment income, and debt

- Relentlessly layer-focused

The reasons we are tempted to allocate capital

- Pricing
- Underwriting
- Line of business performance assessment
- Reinsurance decision making
- Growth planning
- Capital planning
- Etc.

The reasons we are tempted to allocate capital

- Pricing
- Underwriting
- Line of business performance assessment
- Reinsurance decision making
- Growth planning
- Capital planning
- Etc.

- Why?
  - Because we think that allocated capital is relevant to these questions

What we really want to do

- Allocate the ***cost*** of capital
  - Aggregate (pooled) cost usually an input
  - But how to allocate?

What we really want to do

- Allocate the ***cost*** of capital
  - Aggregate (pooled) cost usually an input
  - But how to allocate?

- How we typically do it ("Industry Standard Approach"):
  - Allocate capital,
  - then multiply by (one, unique) hurdle rate.

What we really want to do

- Allocate the ***cost*** of capital
  - Aggregate (pooled) cost usually an input
  - But how to allocate?

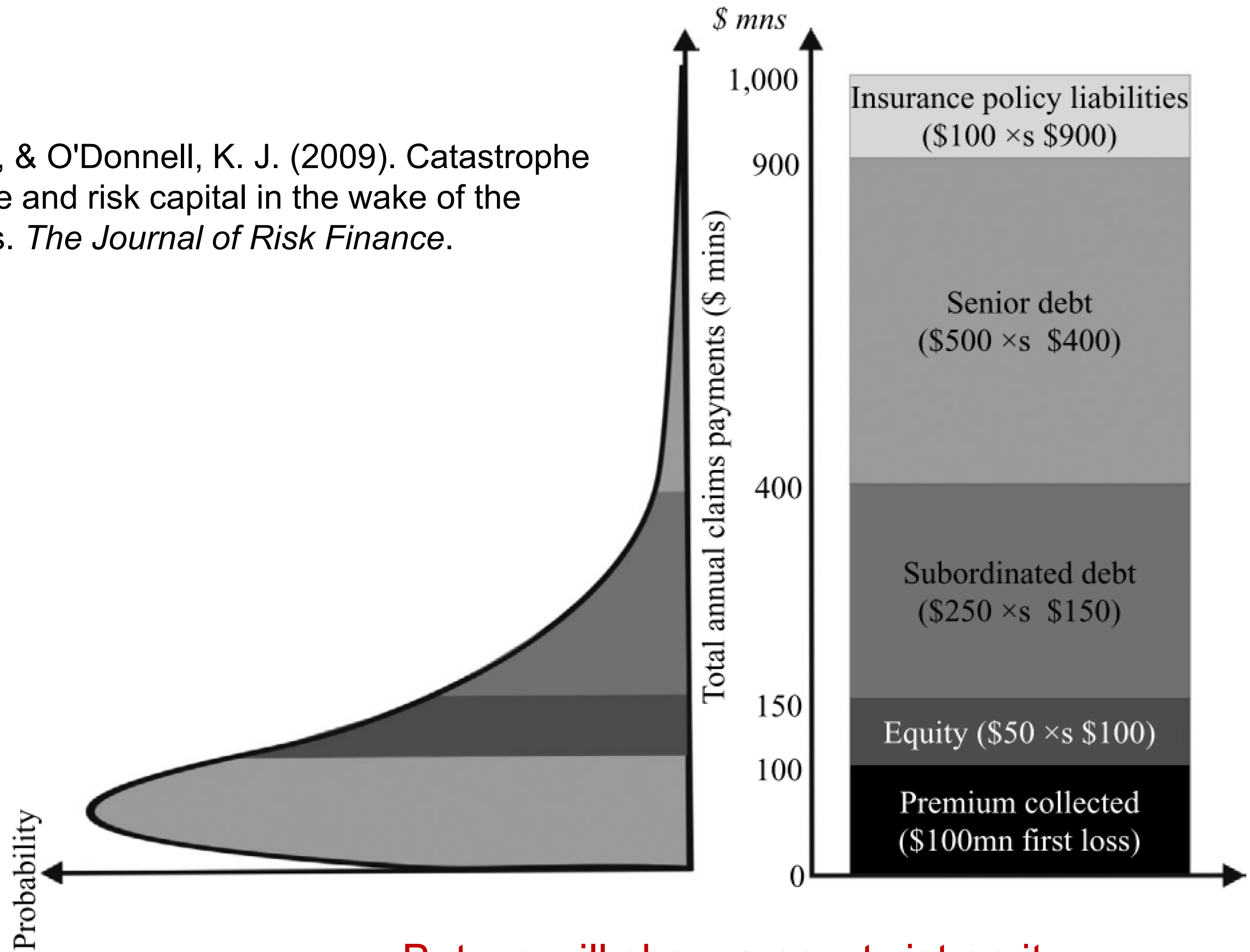
- How we typically do it ("Industry Standard Approach"):
  - Allocate capital,
  - then multiply by (one, unique) hurdle rate.

- Fallacy:
  - "Capital is fungible, so every unit requires the same return"
  - ***FALSE***

This is not a new insight

Culp, C. L., & O'Donnell, K. J. (2009). Catastrophe reinsurance and risk capital in the wake of the credit crisis. *The Journal of Risk Finance*.

Figure 3



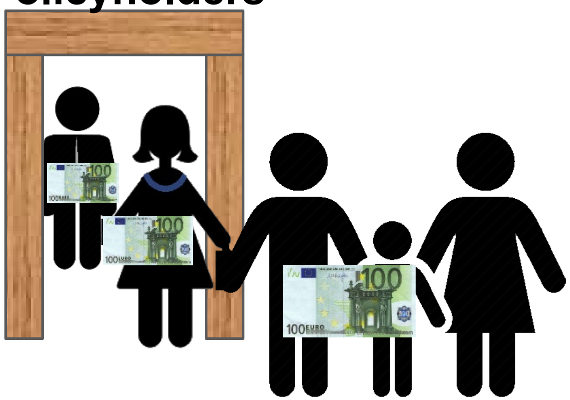
But we will show a new twist on it

How we usually think about operations: (1) funding assets

NOTE:

- *Premium is net of expenses*
- *Claims are net of expenses*
- *Risk-free rate is 0*

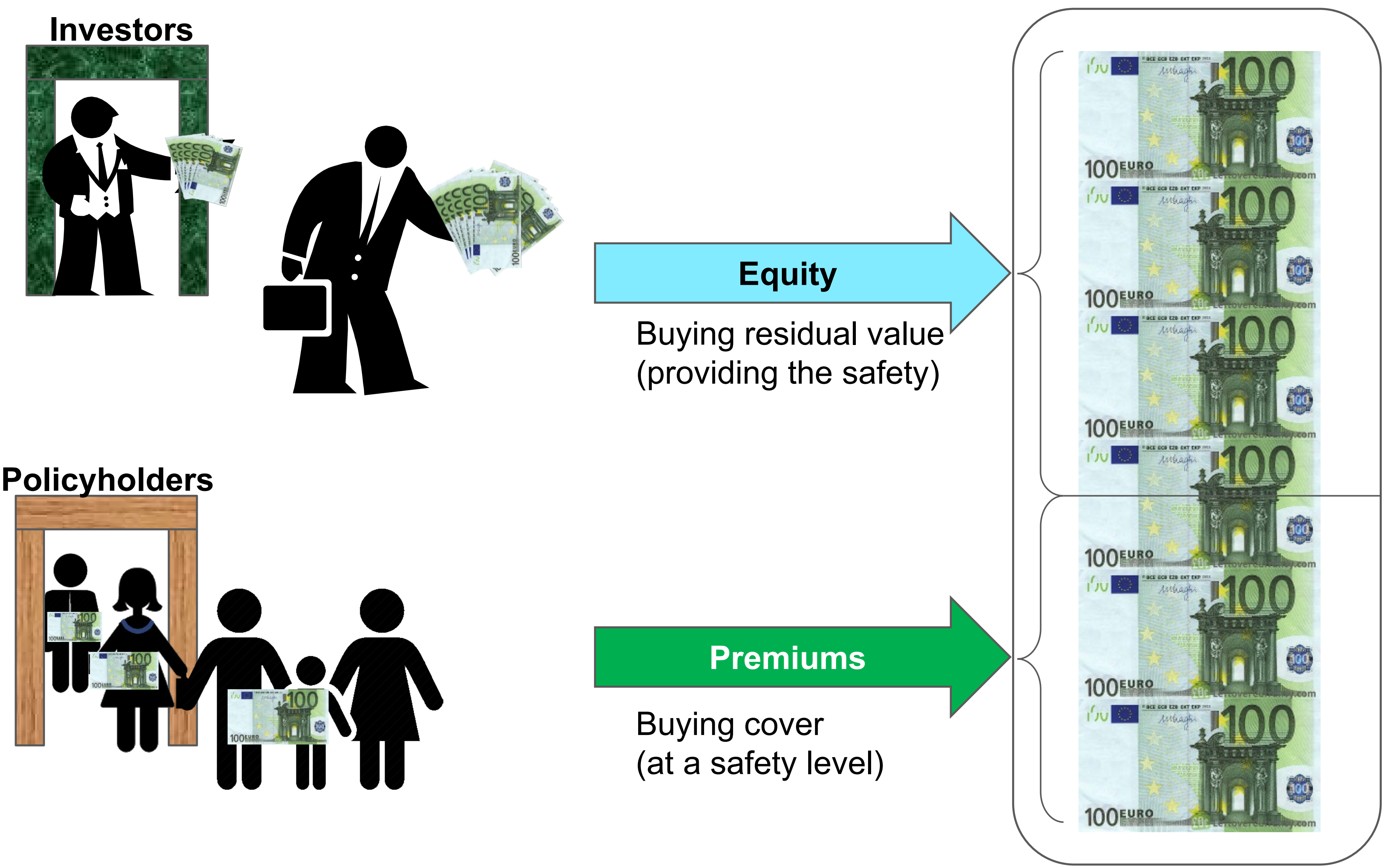
Policyholders



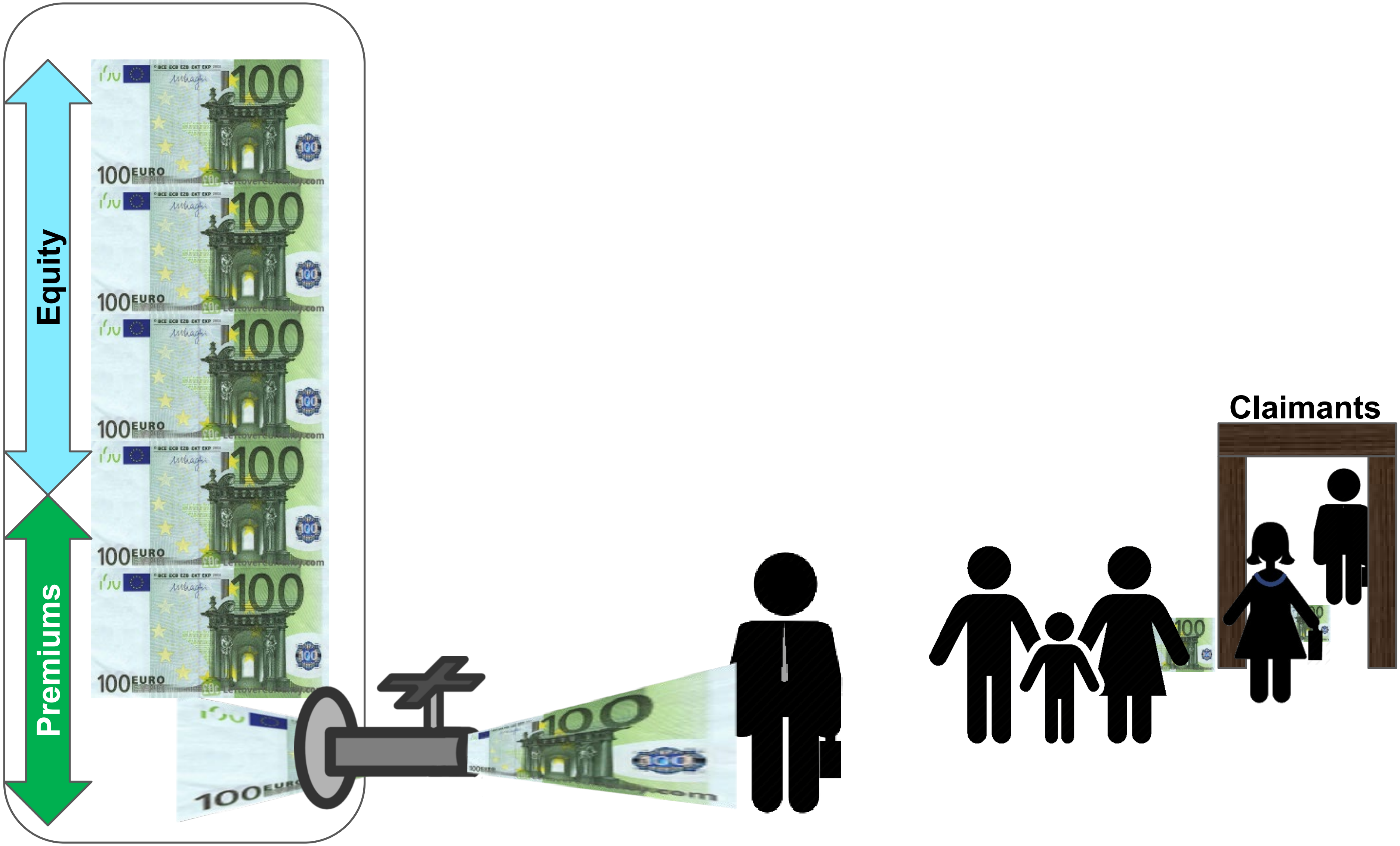
Premiums

Buying cover
(at a safety level)

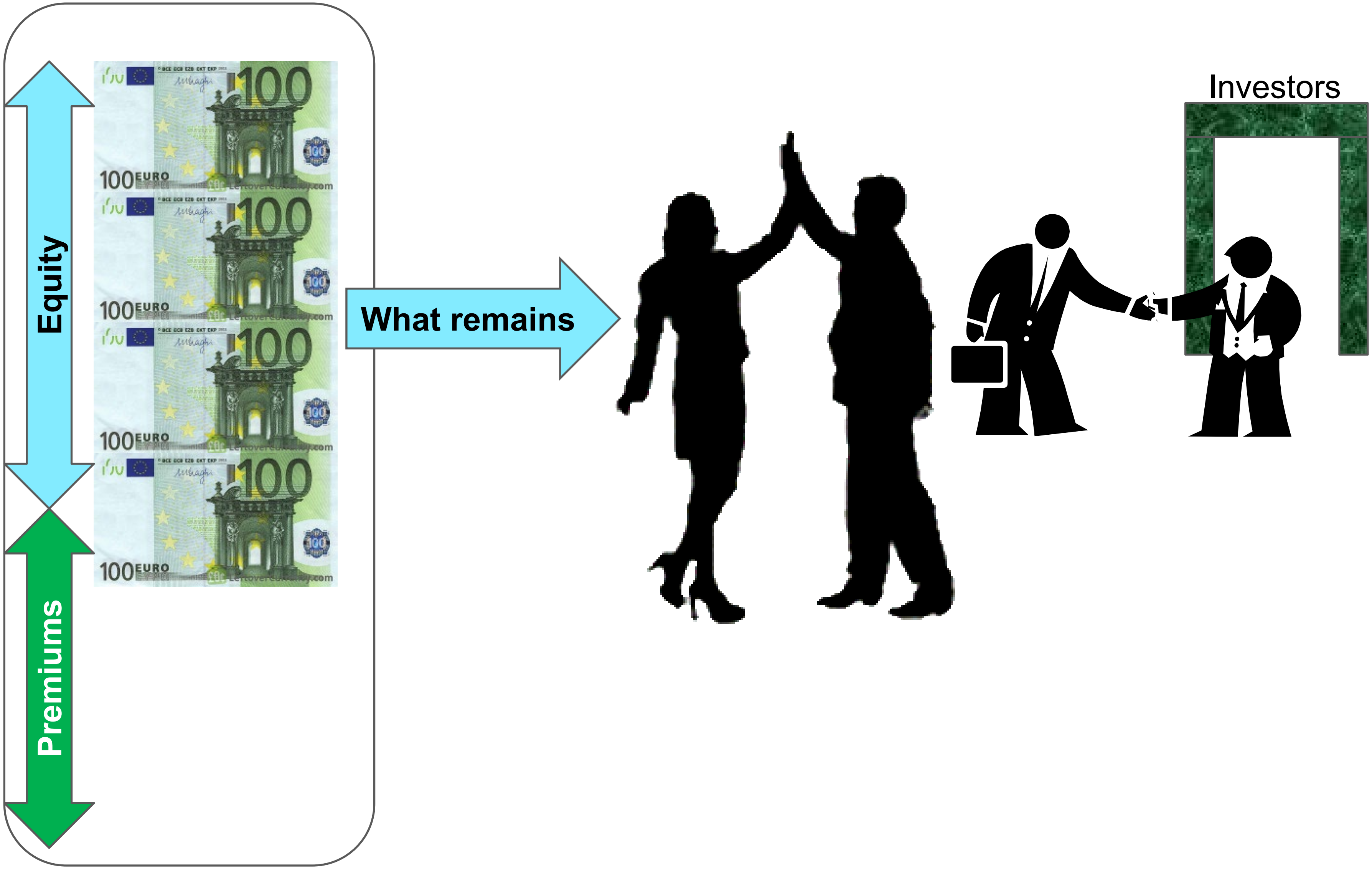
How we usually think about operations: (1) funding assets



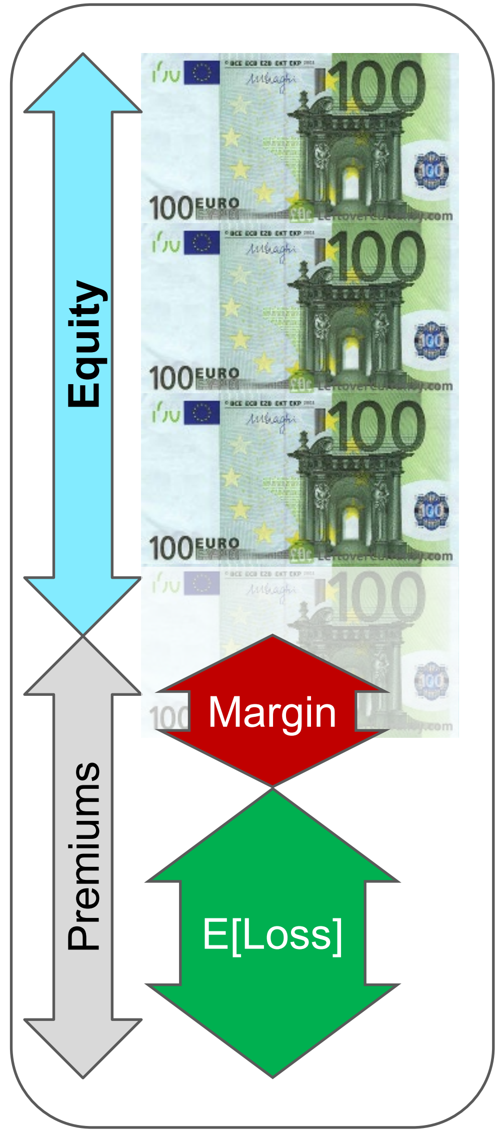
How we usually think about operations: (2) paying claims



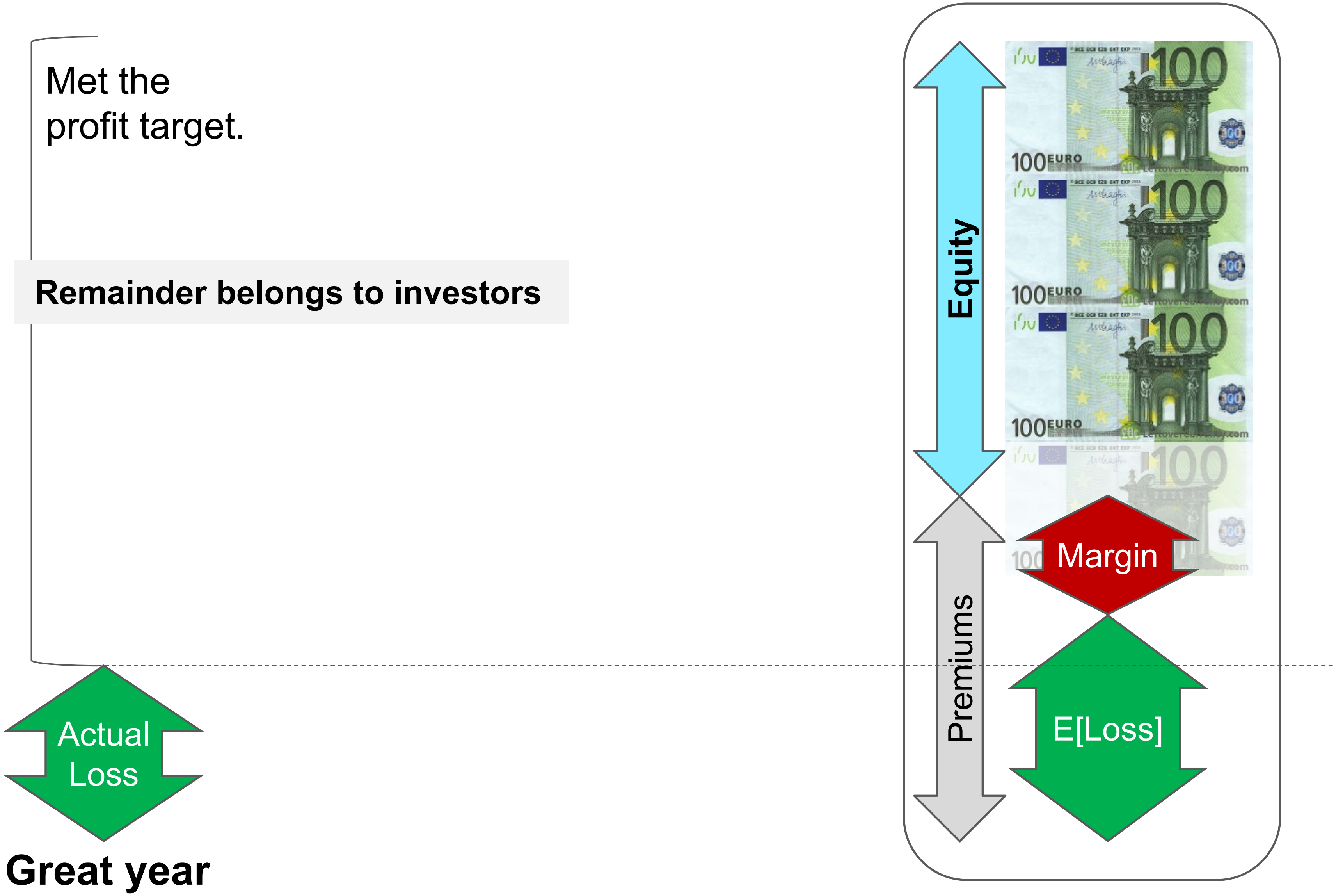
How we usually think about operations: (3) residual value



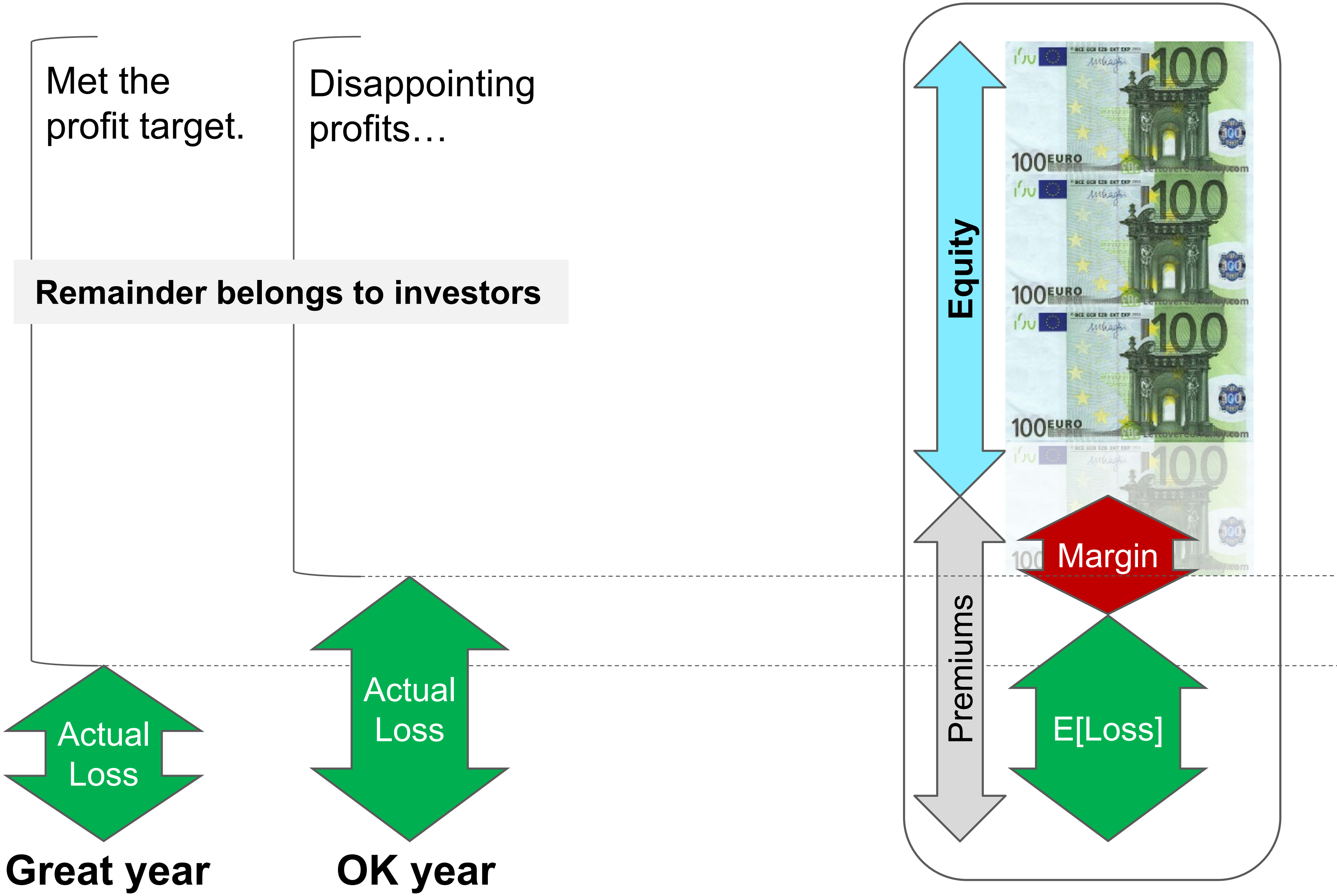
How we usually think about operations: (4) financial reporting



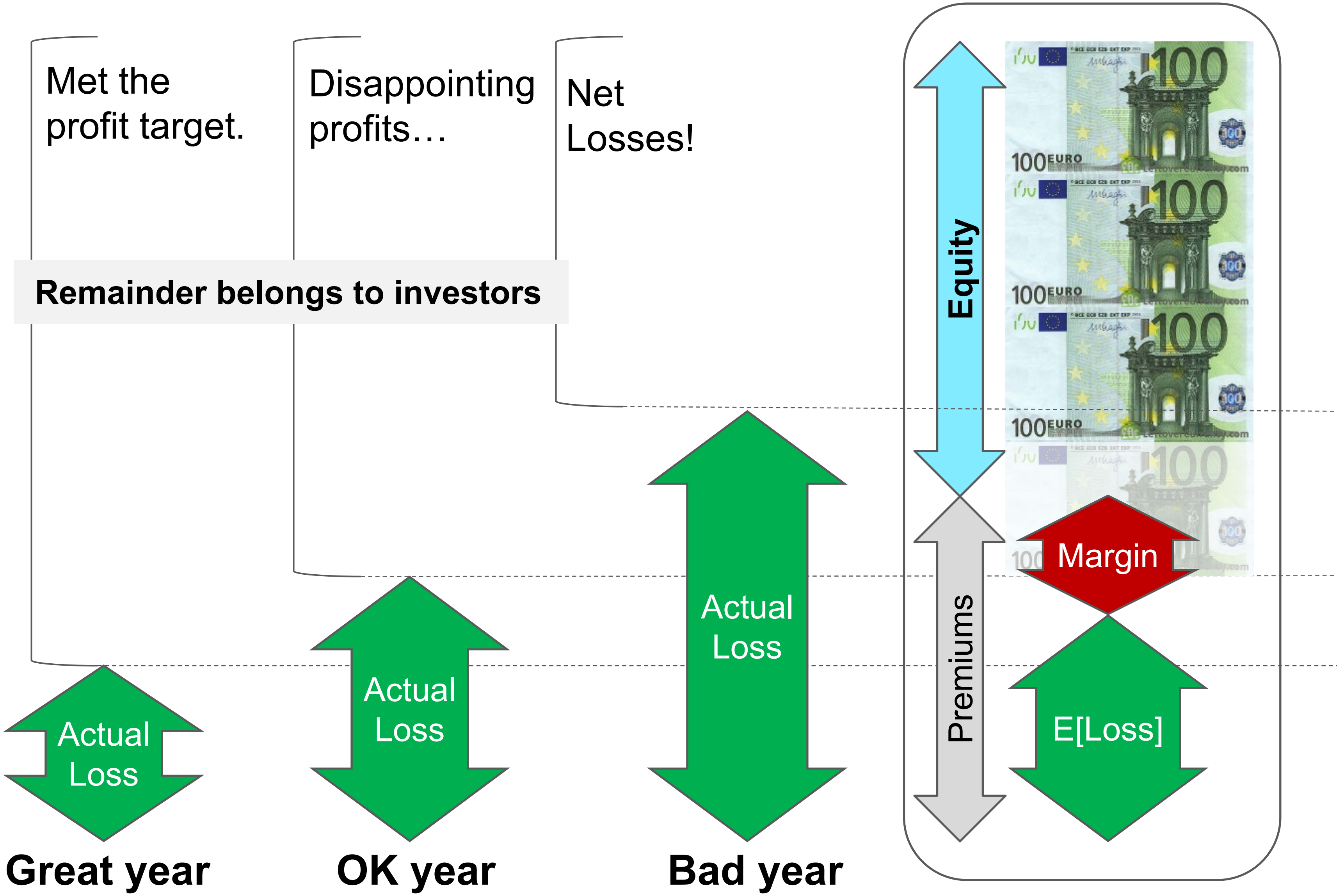
How we usually think about operations: (4) financial reporting



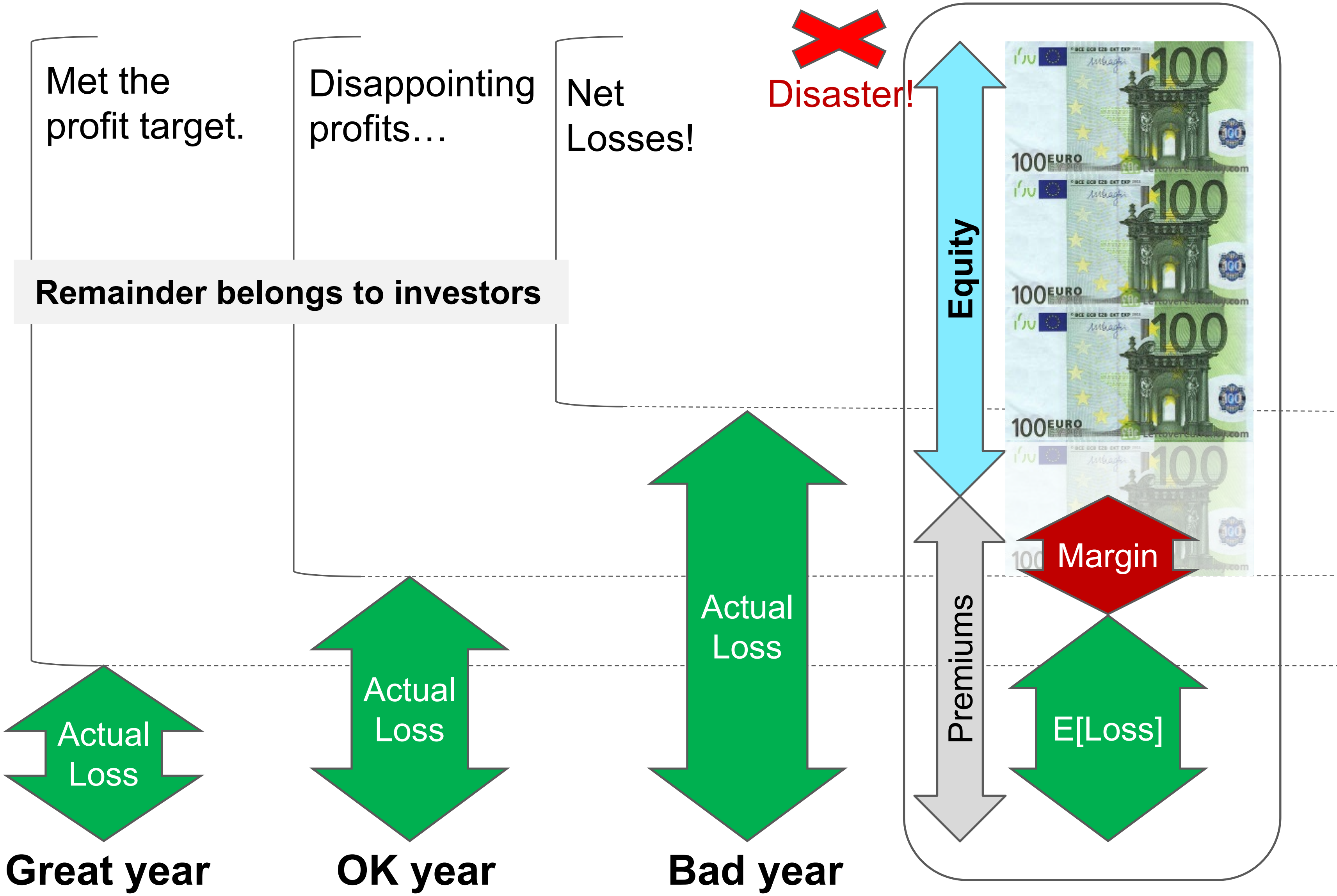
How we usually think about operations: (4) financial reporting



How we usually think about operations: (4) financial reporting



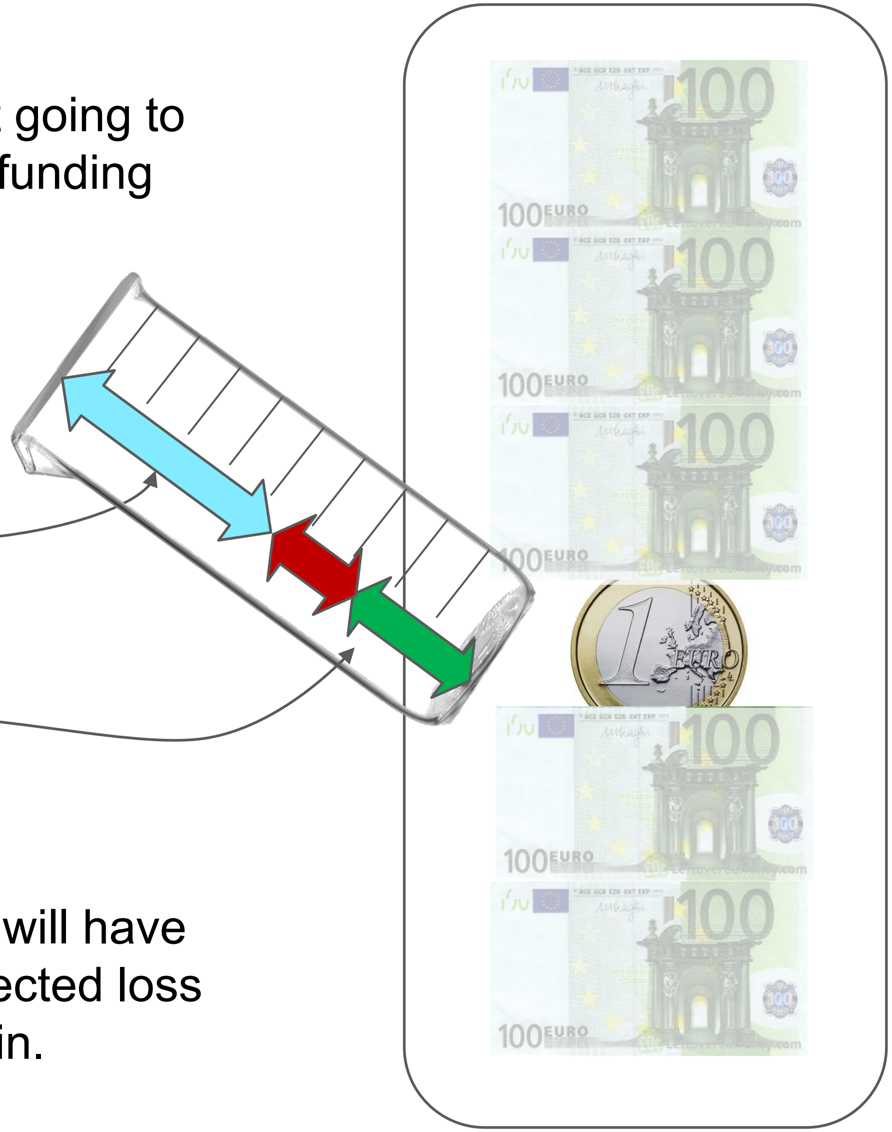
How we usually think about operations: (4) financial reporting



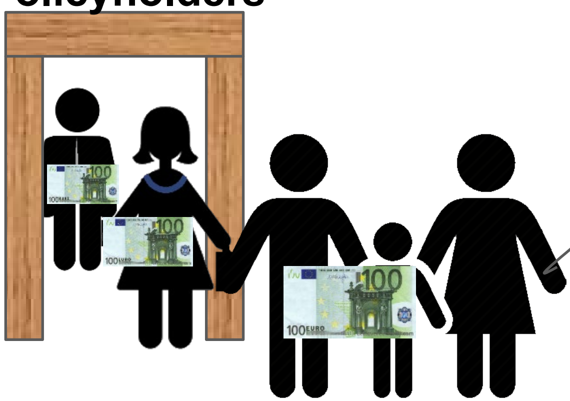
What if you had to fund each asset unit (layer) individually?

Investors

Investor is not going to supply *all* the funding



Policyholders



Policyholders will have to supply expected loss and risk margin.

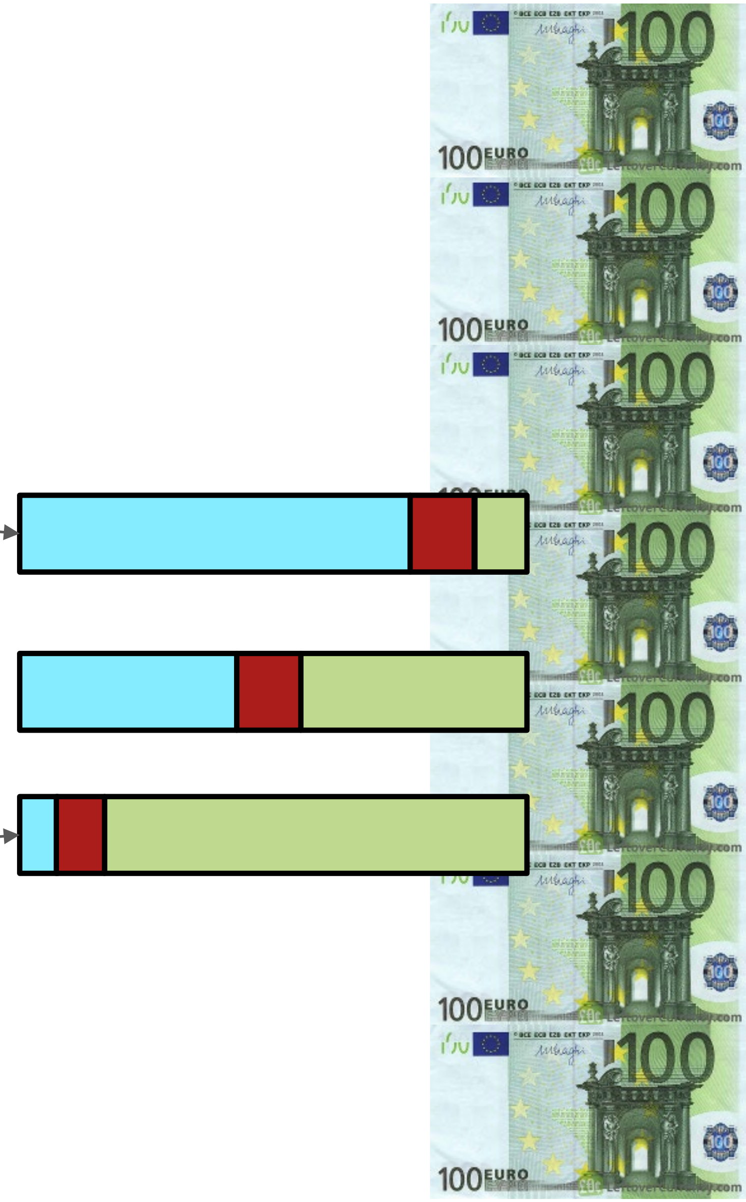
Every unit (layer) of ***asset*** has expected loss, risk margin, and equity

- Attach @ high loss
- Low probability
- High residual value

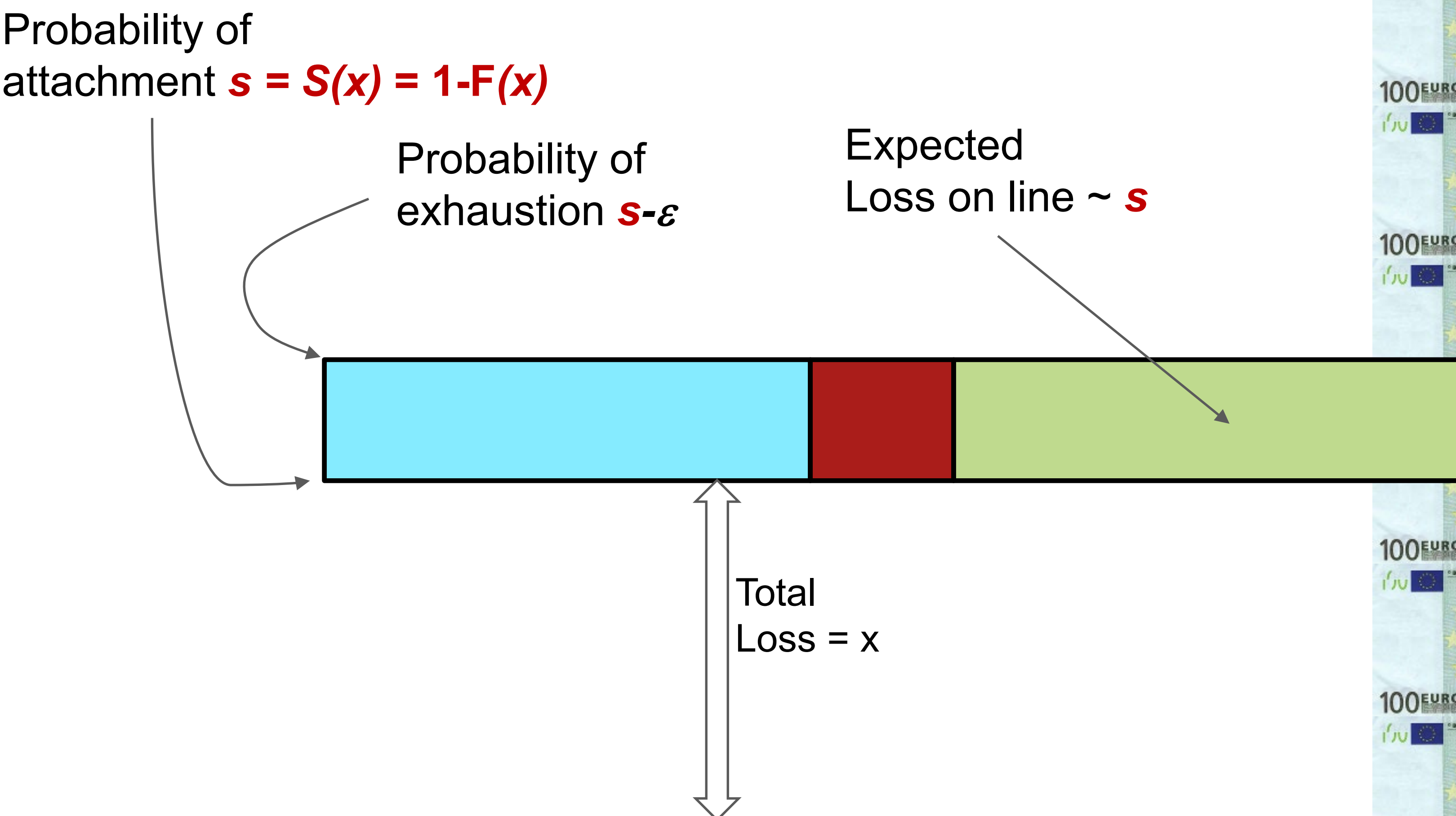
- Low premium
- High equity

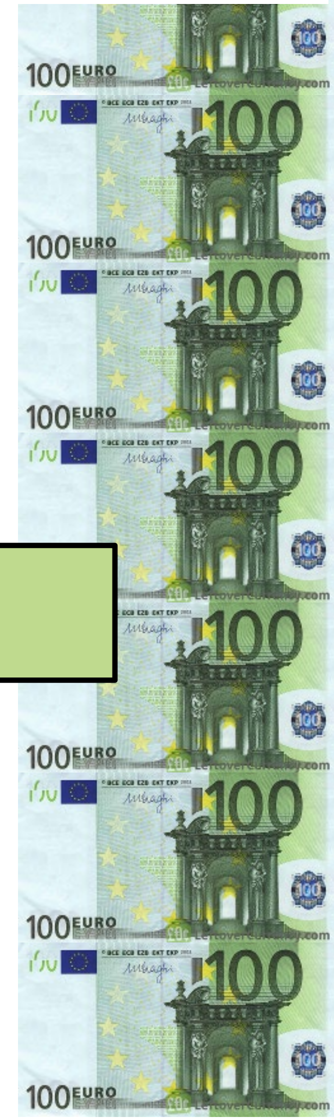
- Attach @ low loss
- High probability
- Low residual value

- High premium
- Low equity

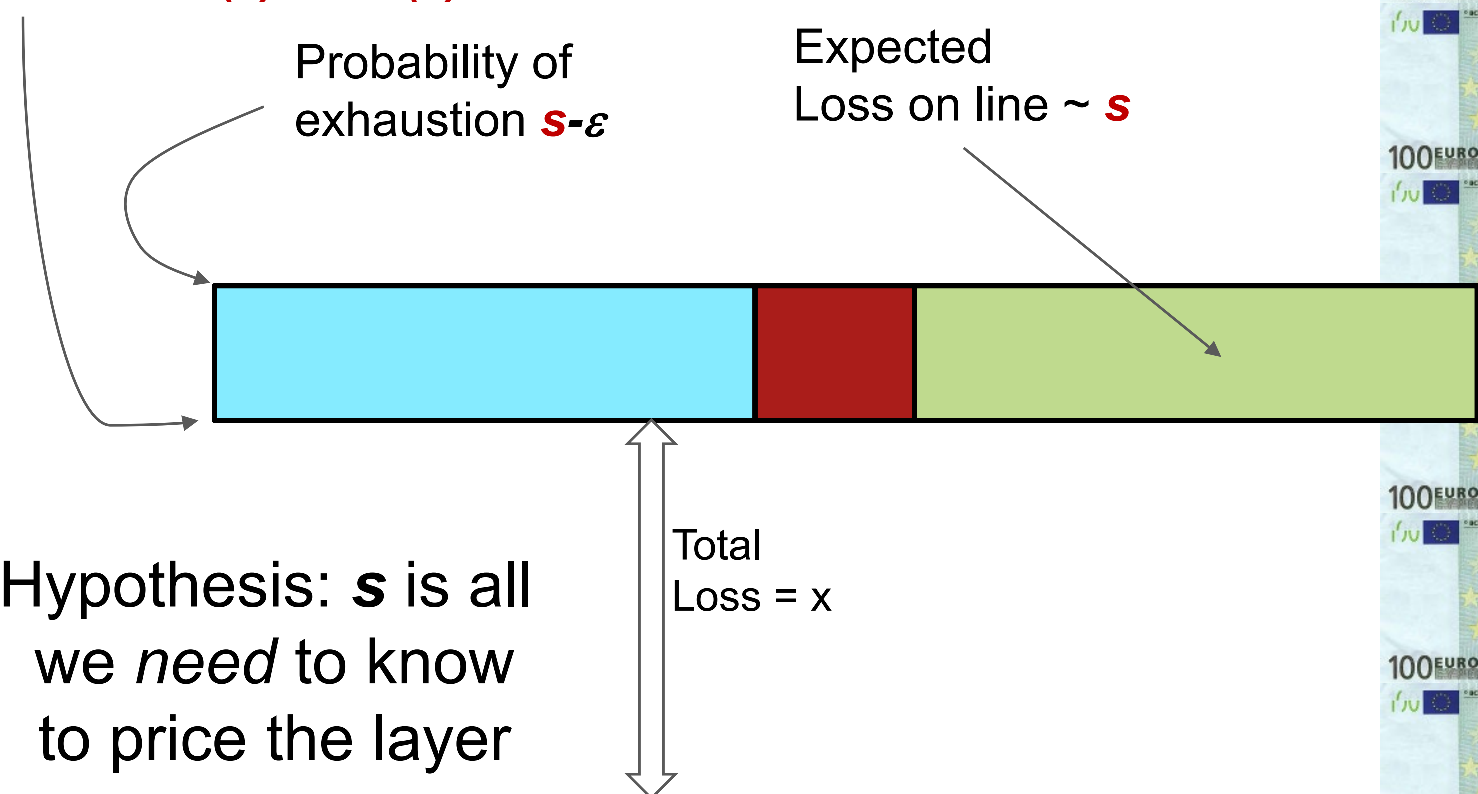


What do we know about a thin layer on the portfolio aggregate loss?

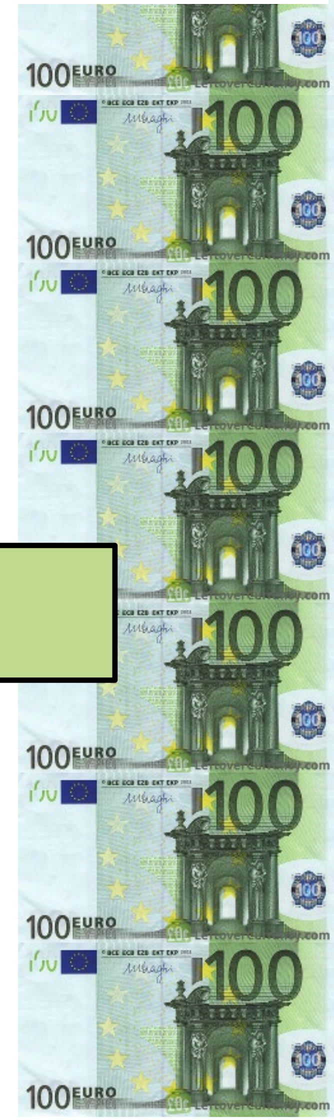




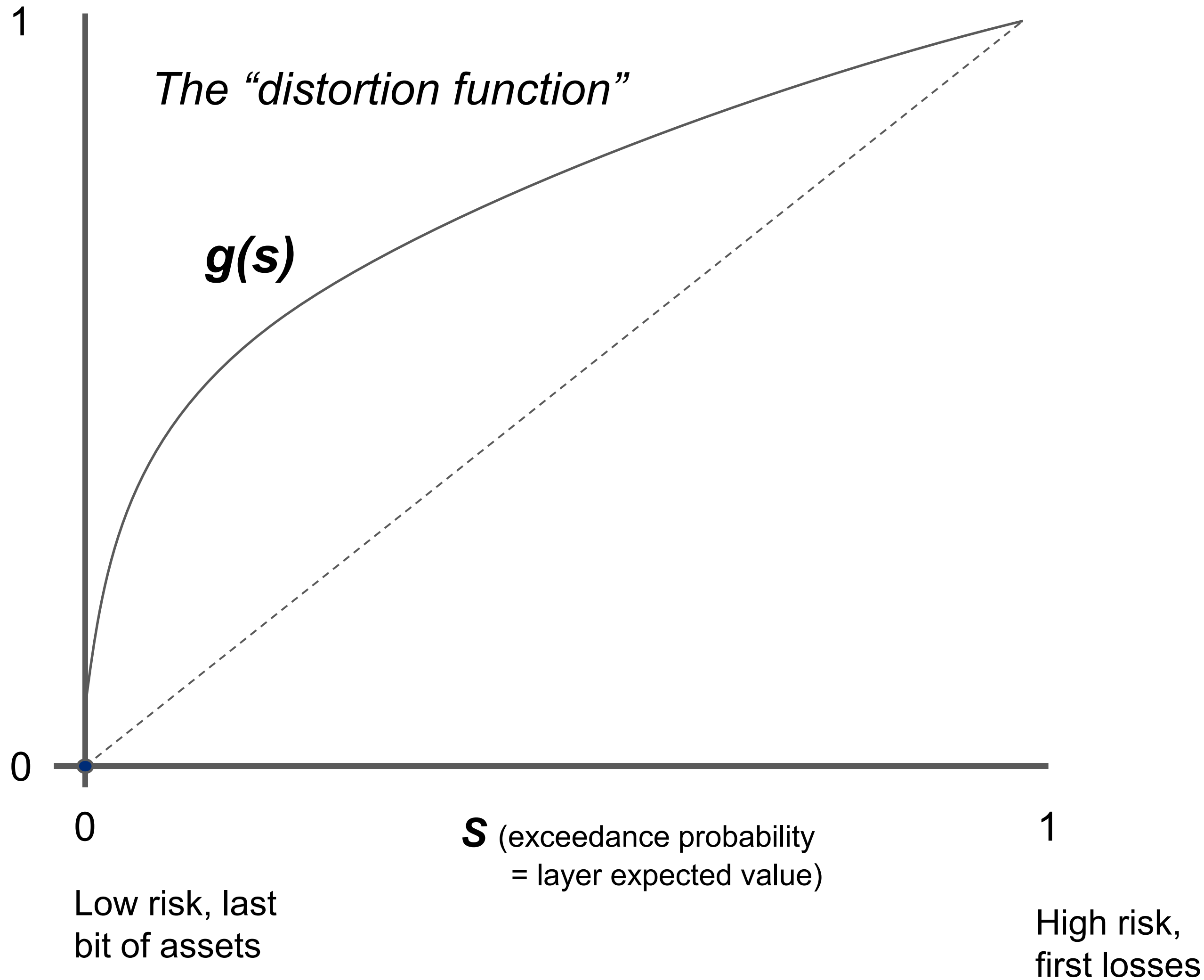
What do we know about a thin layer on the portfolio aggregate loss?



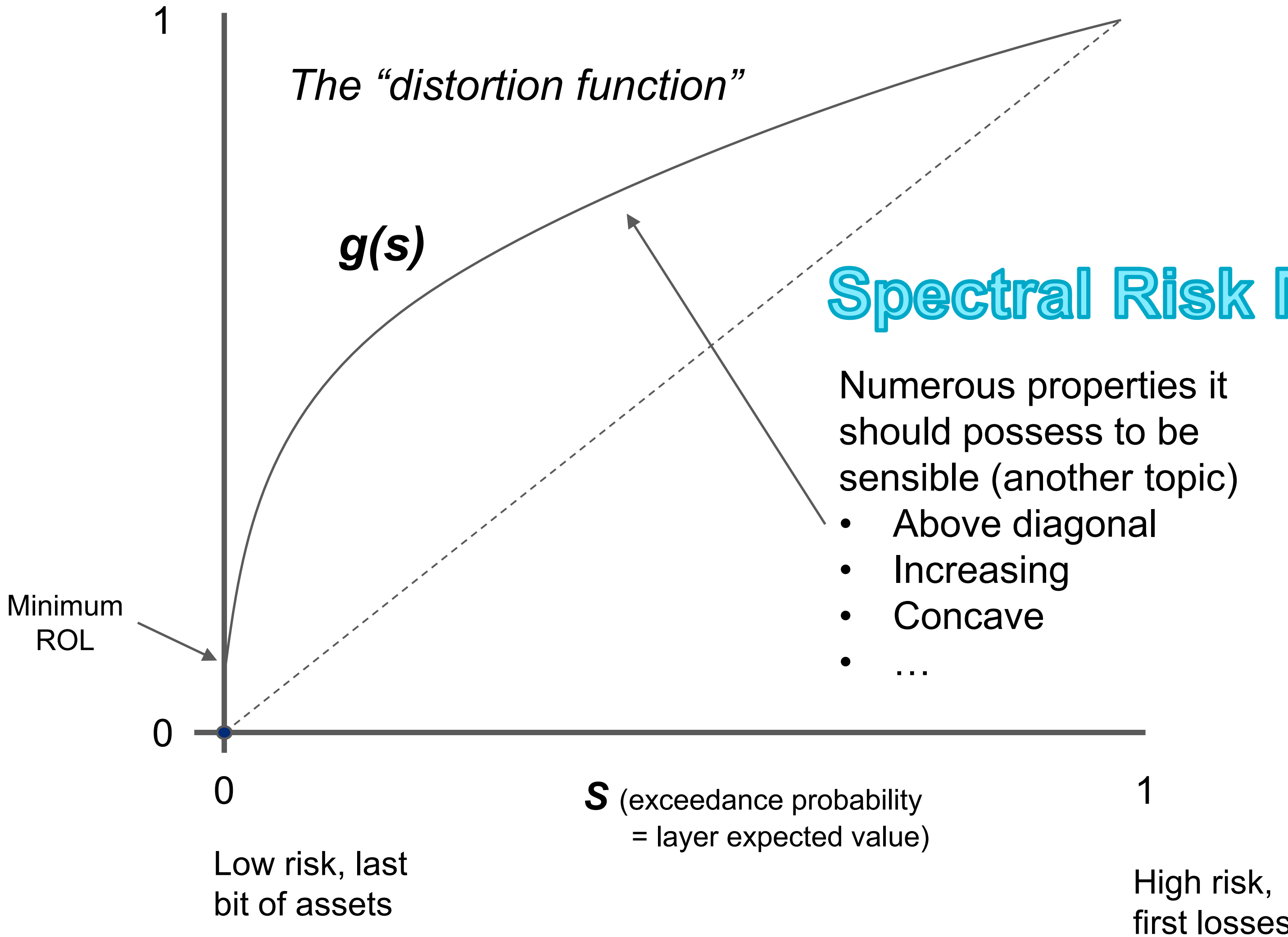
Hypothesis: ***s*** is all we *need* to know to price the layer



Assumption: there is a functional relationship between layer LOL and layer ROL



Assumption: there is a functional relationship between layer LOL and layer ROL

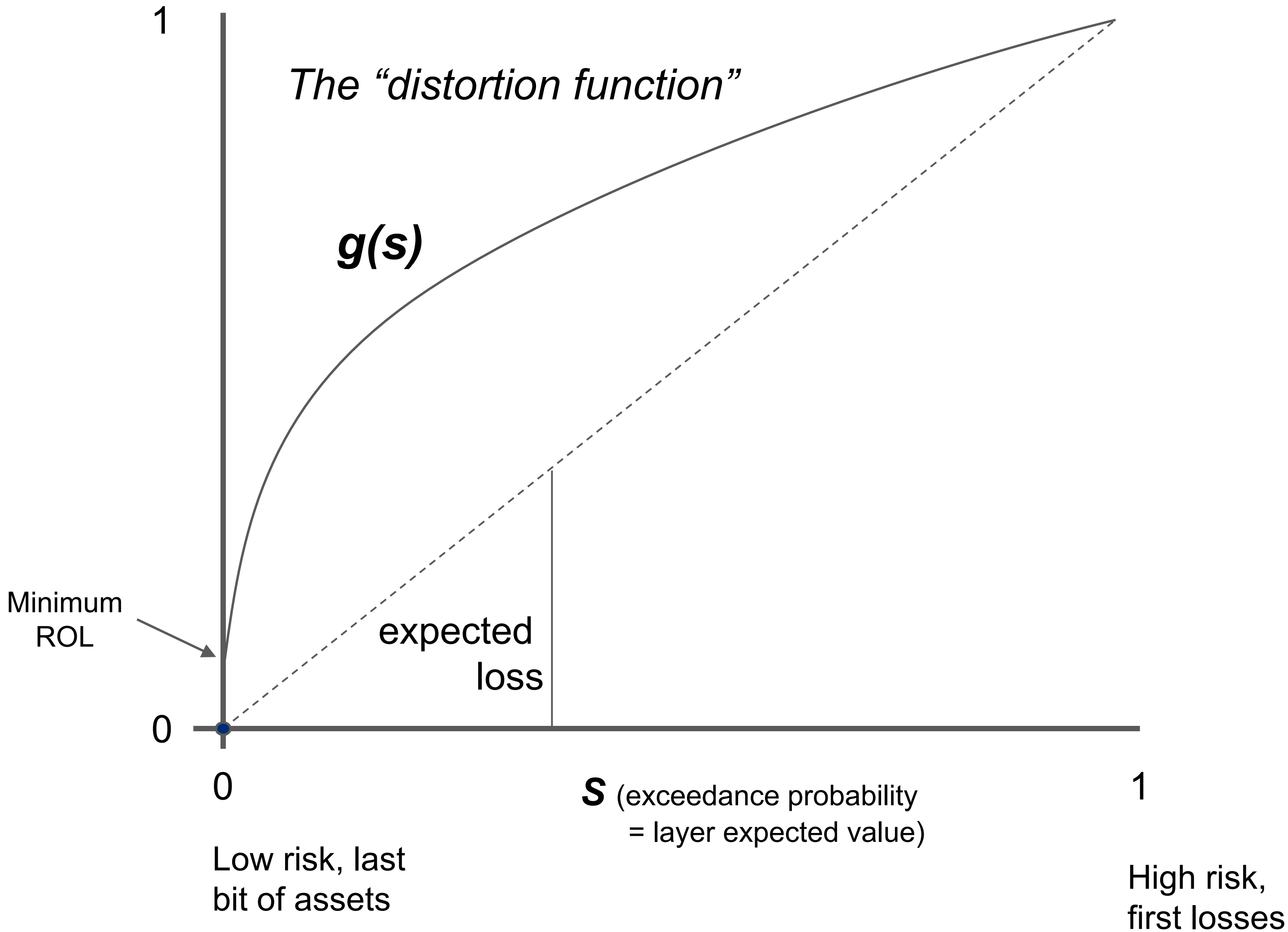


Spectral Risk Measure

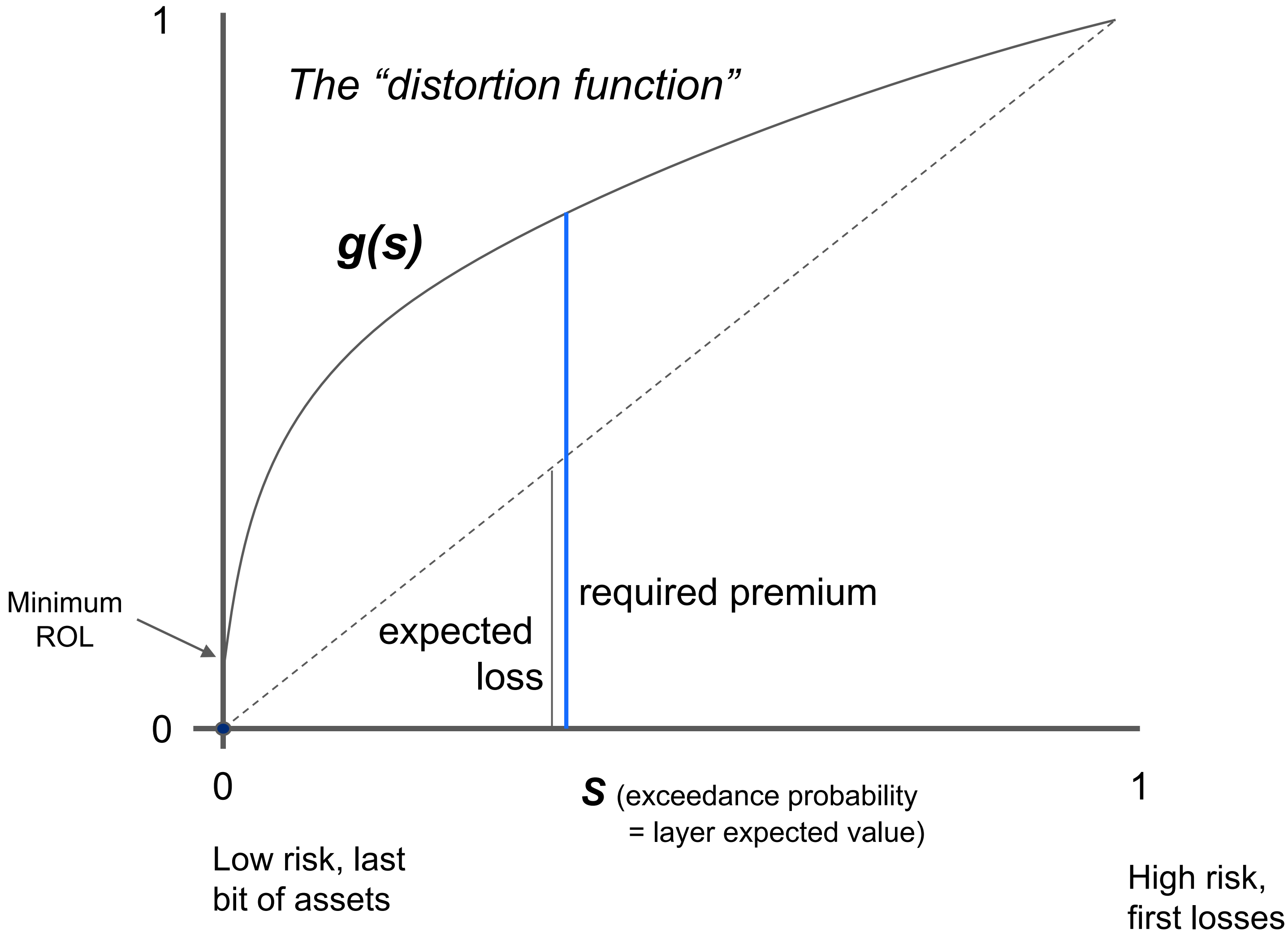
Numerous properties it should possess to be sensible (another topic)

- Above diagonal
- Increasing
- Concave
- …

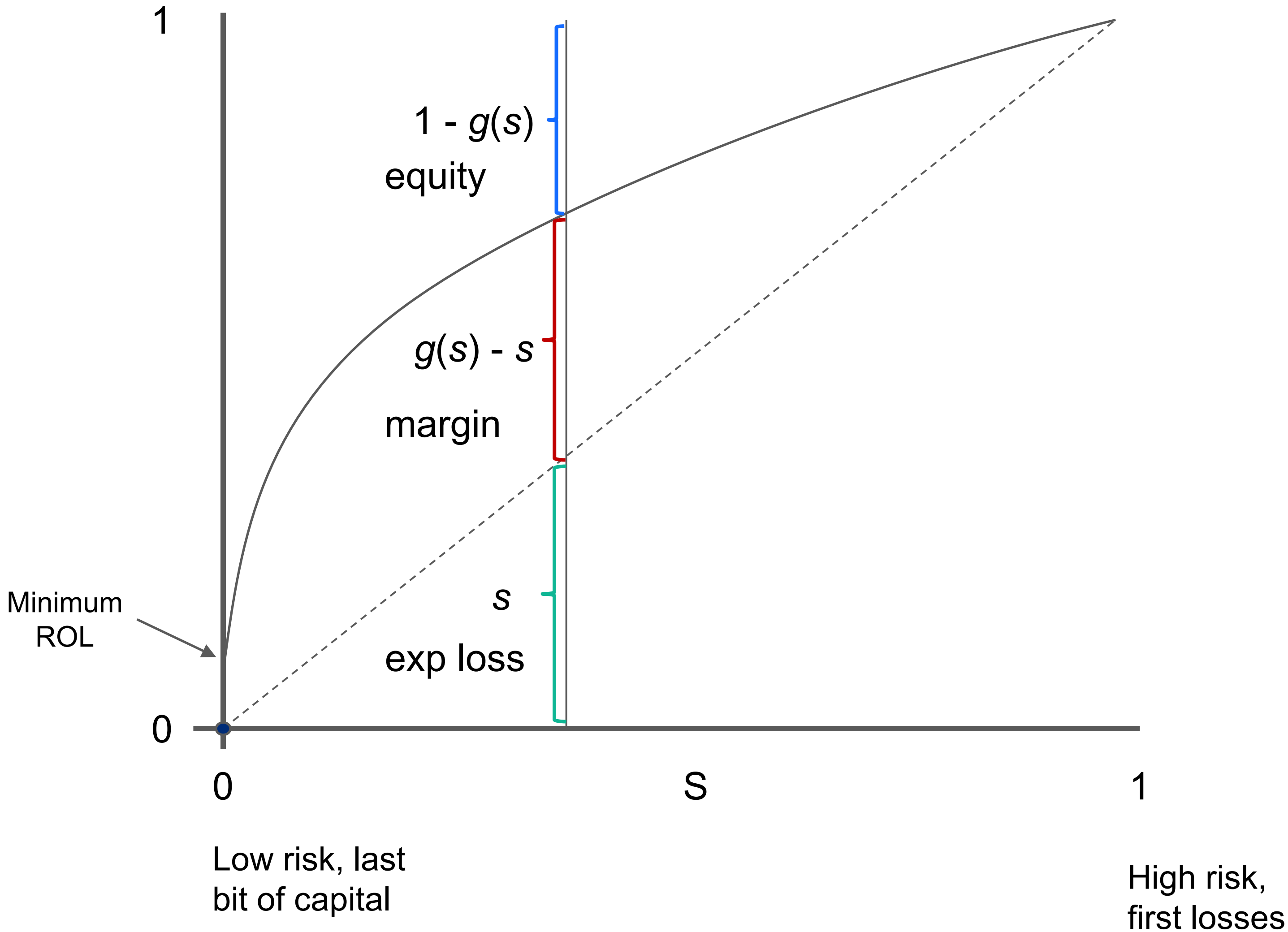
Assumption: there is a functional relationship between layer LOL and layer ROL



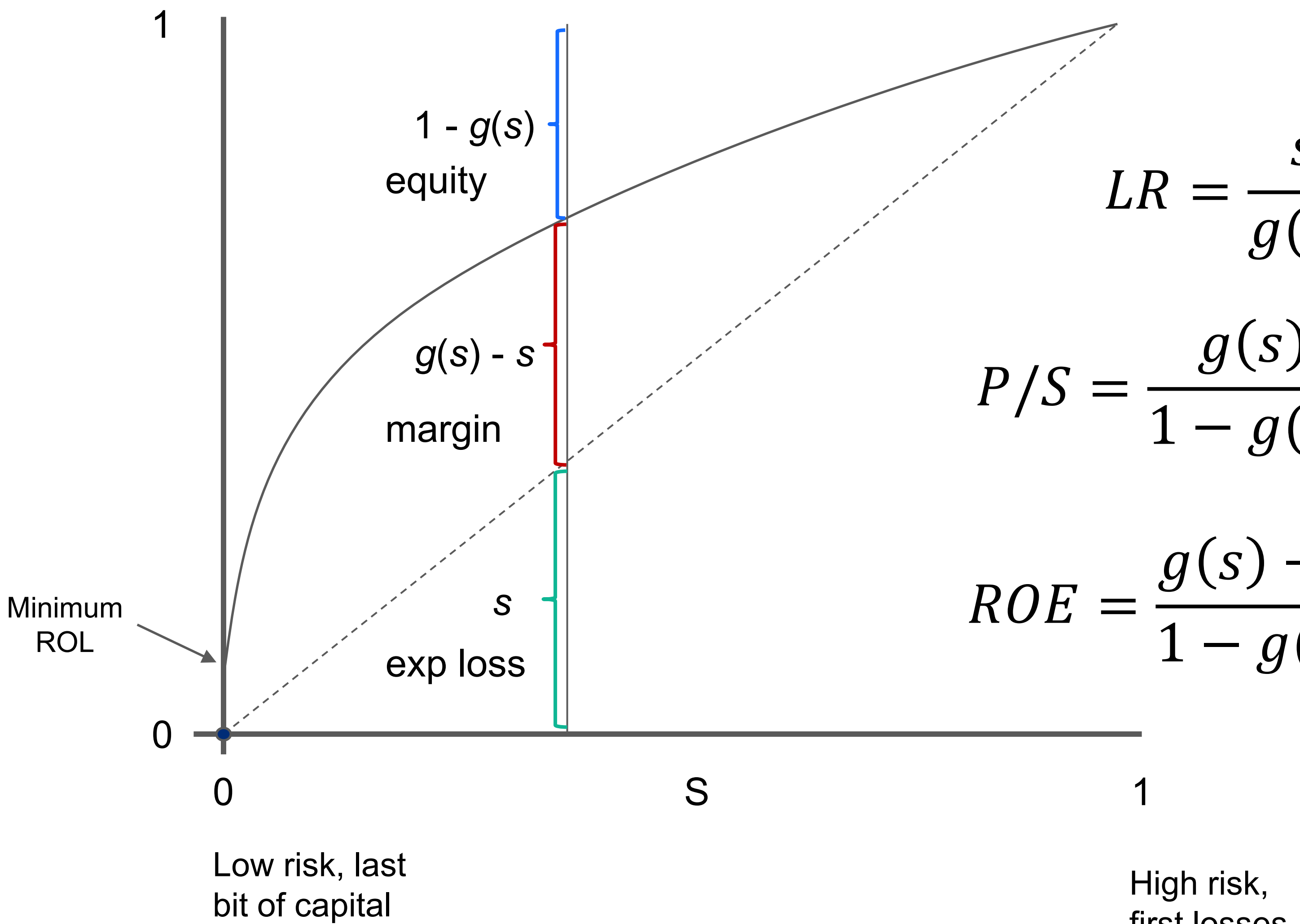
Assumption: there is a functional relationship between layer LOL and layer ROL



Distortion function gives you everything you want to know



Distortion function gives you everything you want to know

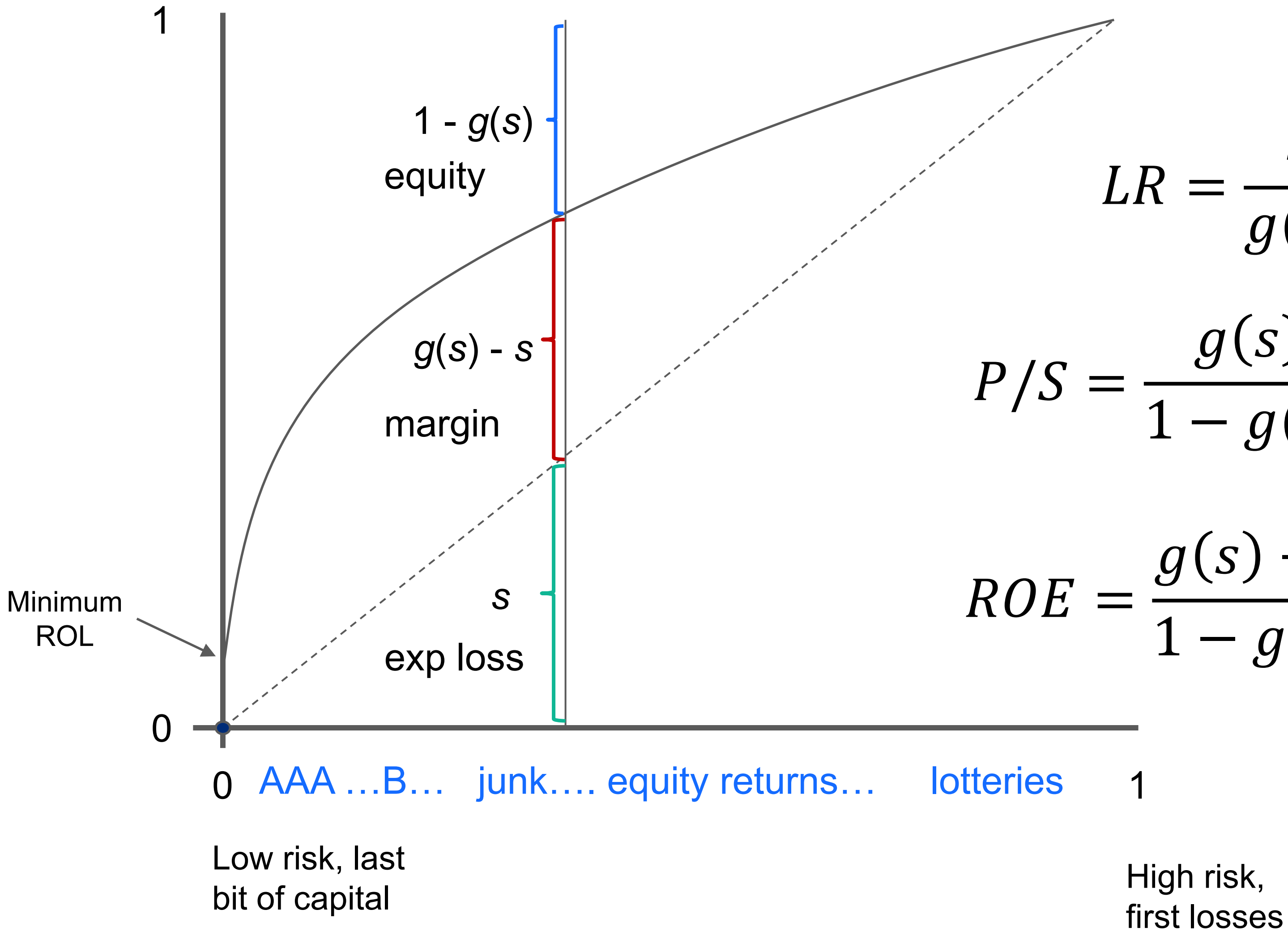


$$LR = \frac{s}{g(s)}$$

$$P/S = \frac{g(s)}{1 - g(s)}$$

$$ROE = \frac{g(s) - s}{1 - g(s)}$$

Distortion function gives you everything you want to know

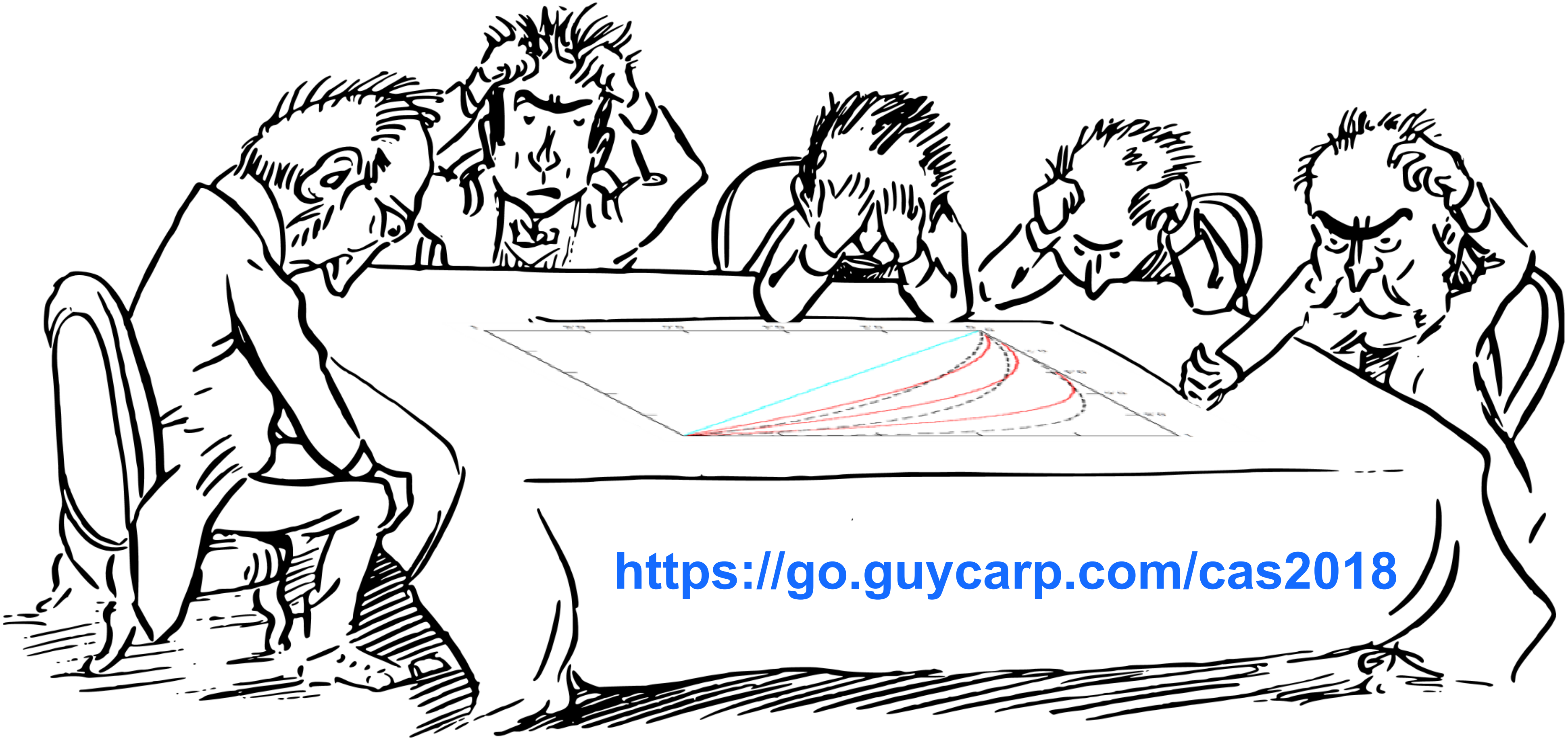


$$LR = \frac{s}{g(s)}$$

$$P/S = \frac{g(s)}{1 - g(s)}$$

$$ROE = \frac{g(s) - s}{1 - g(s)}$$

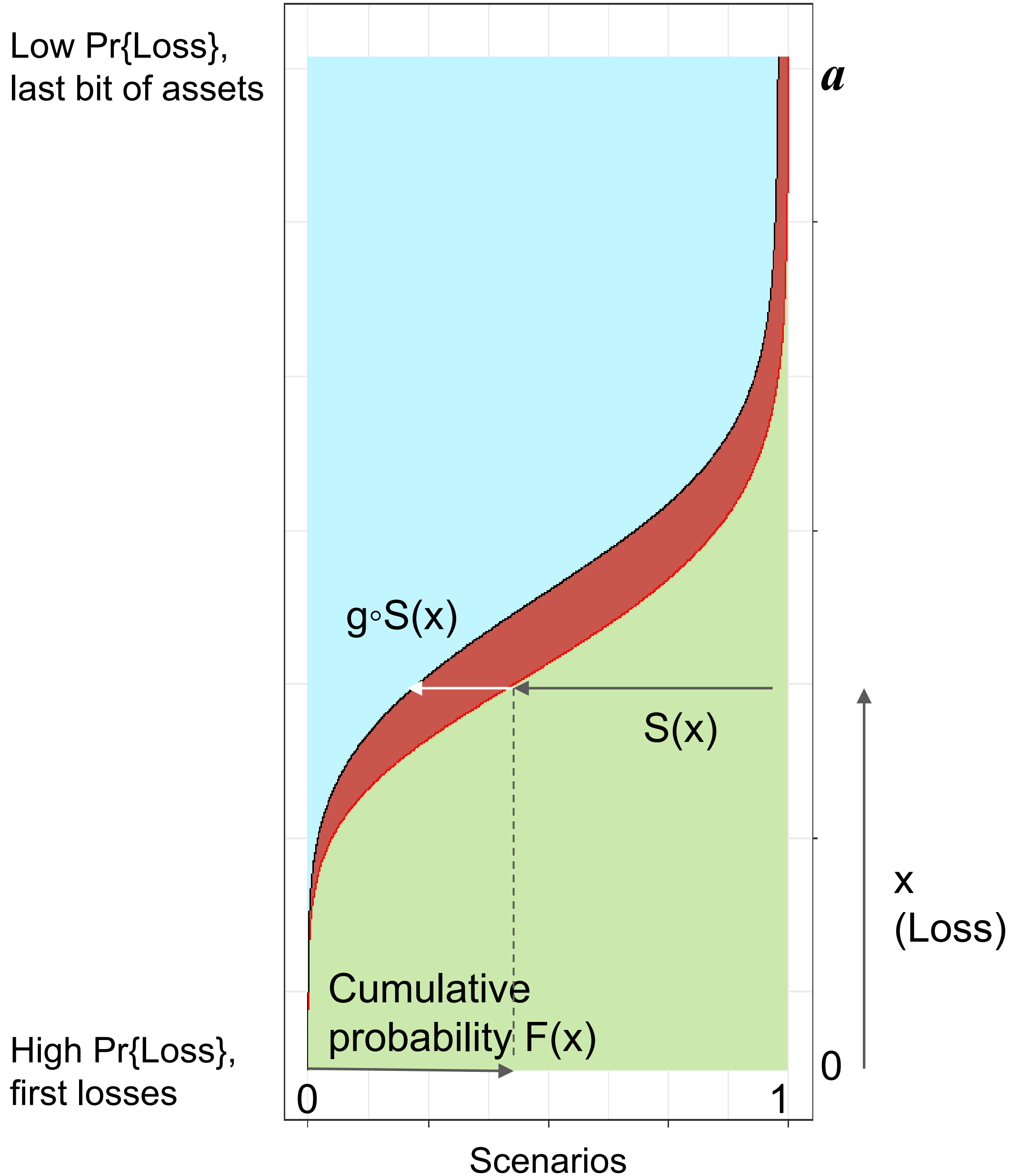
Where does that *g*(*s*) function come from?



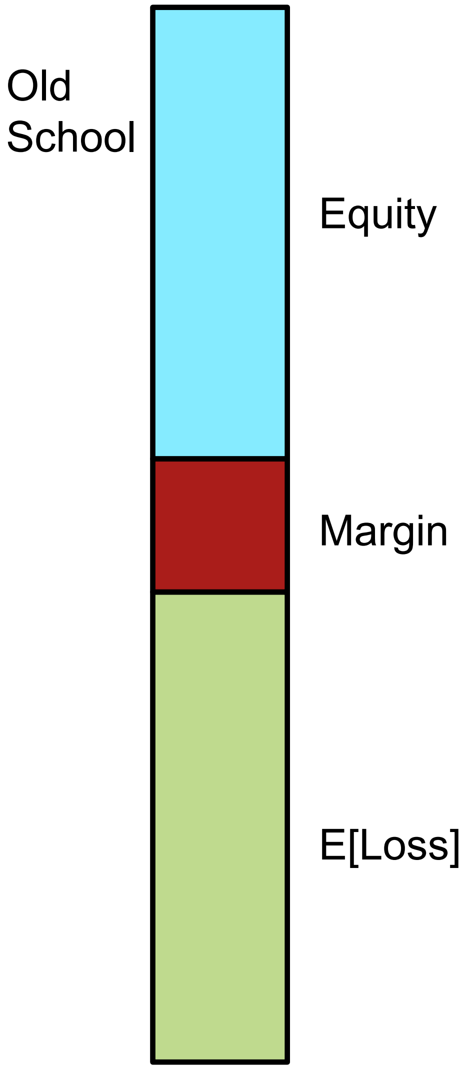
Short answer: “Financial Considerations”

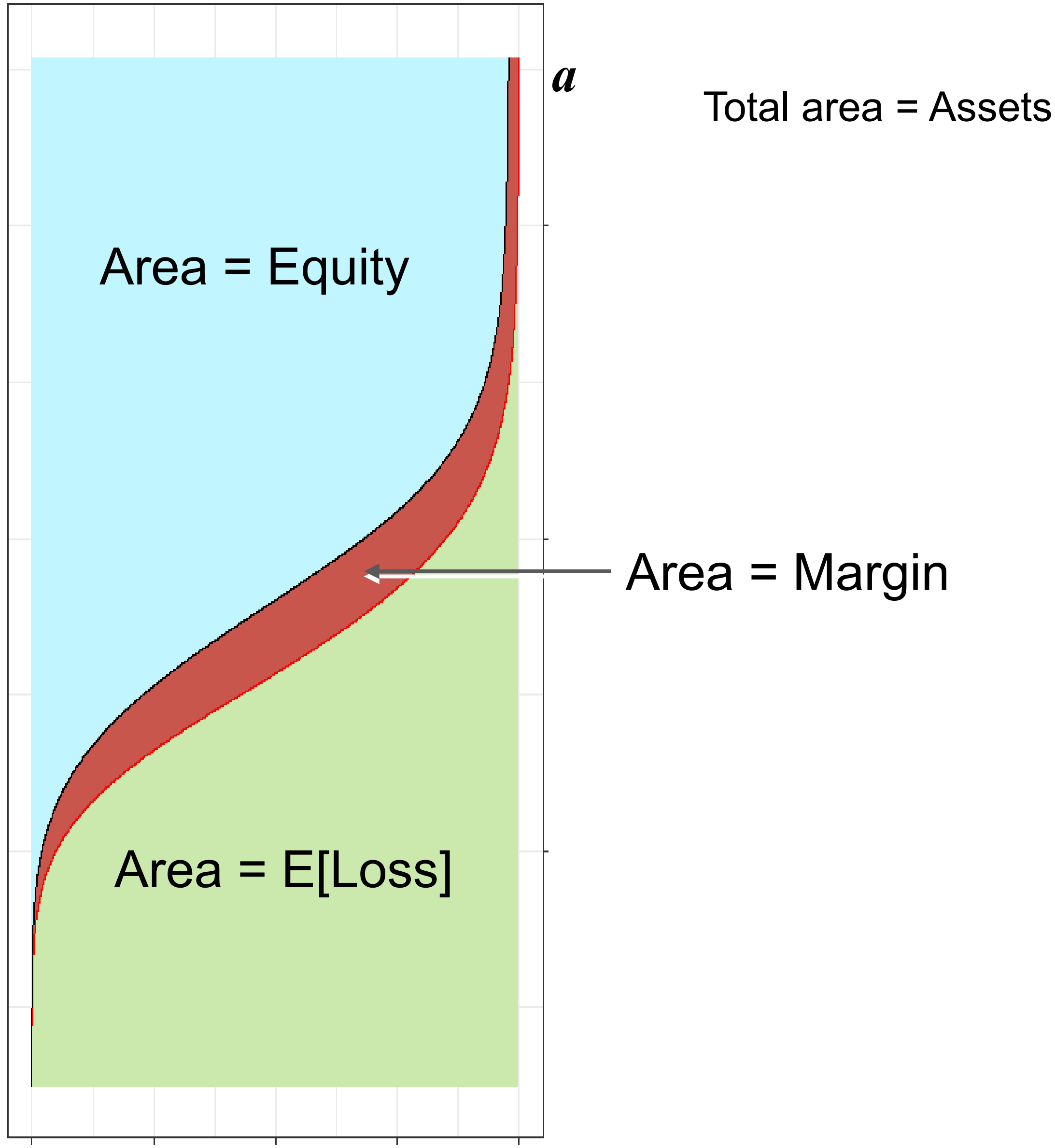
(another topic)

How it looks back in the scenario-loss domain

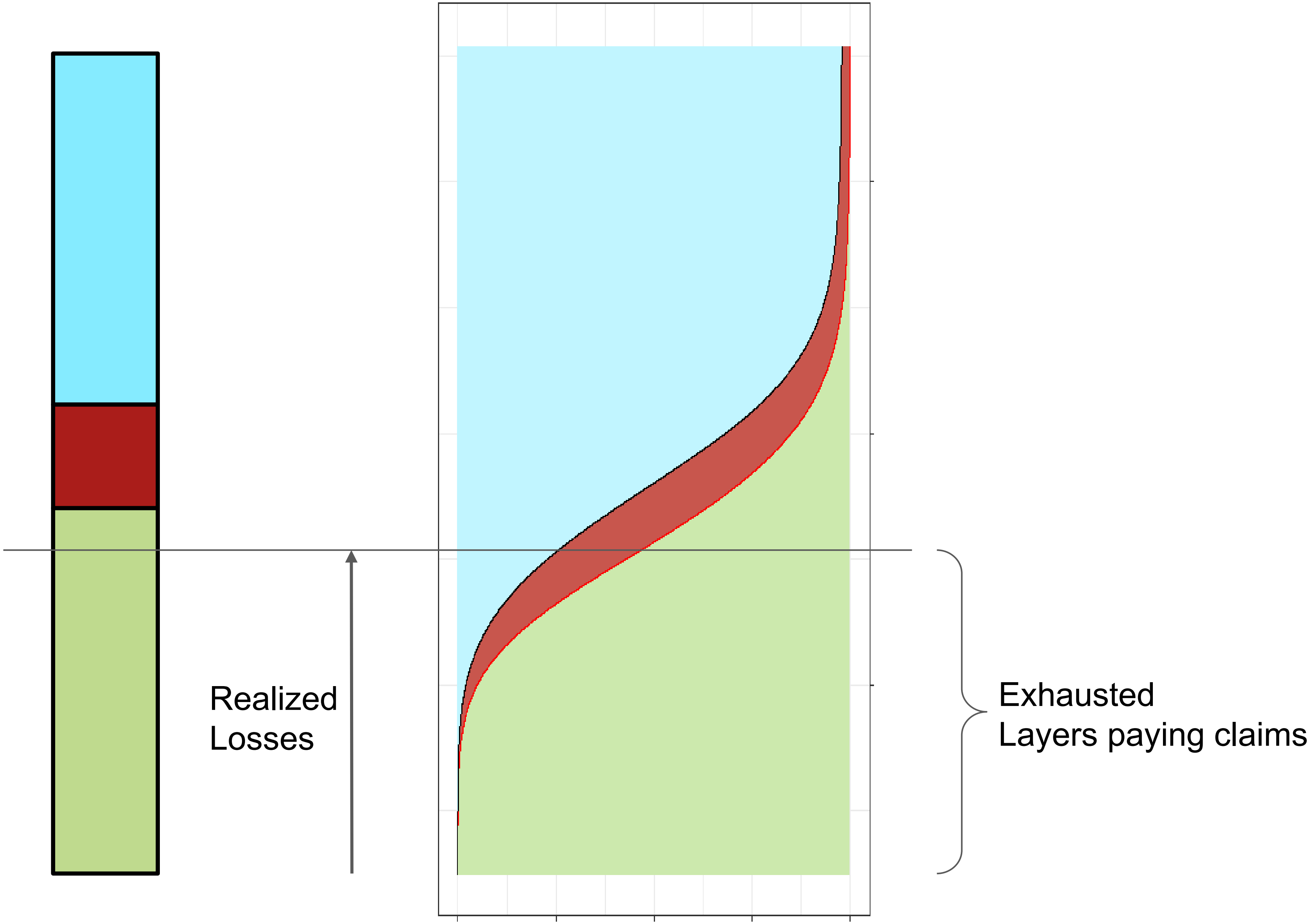


The new perspective on where premium and equity sit

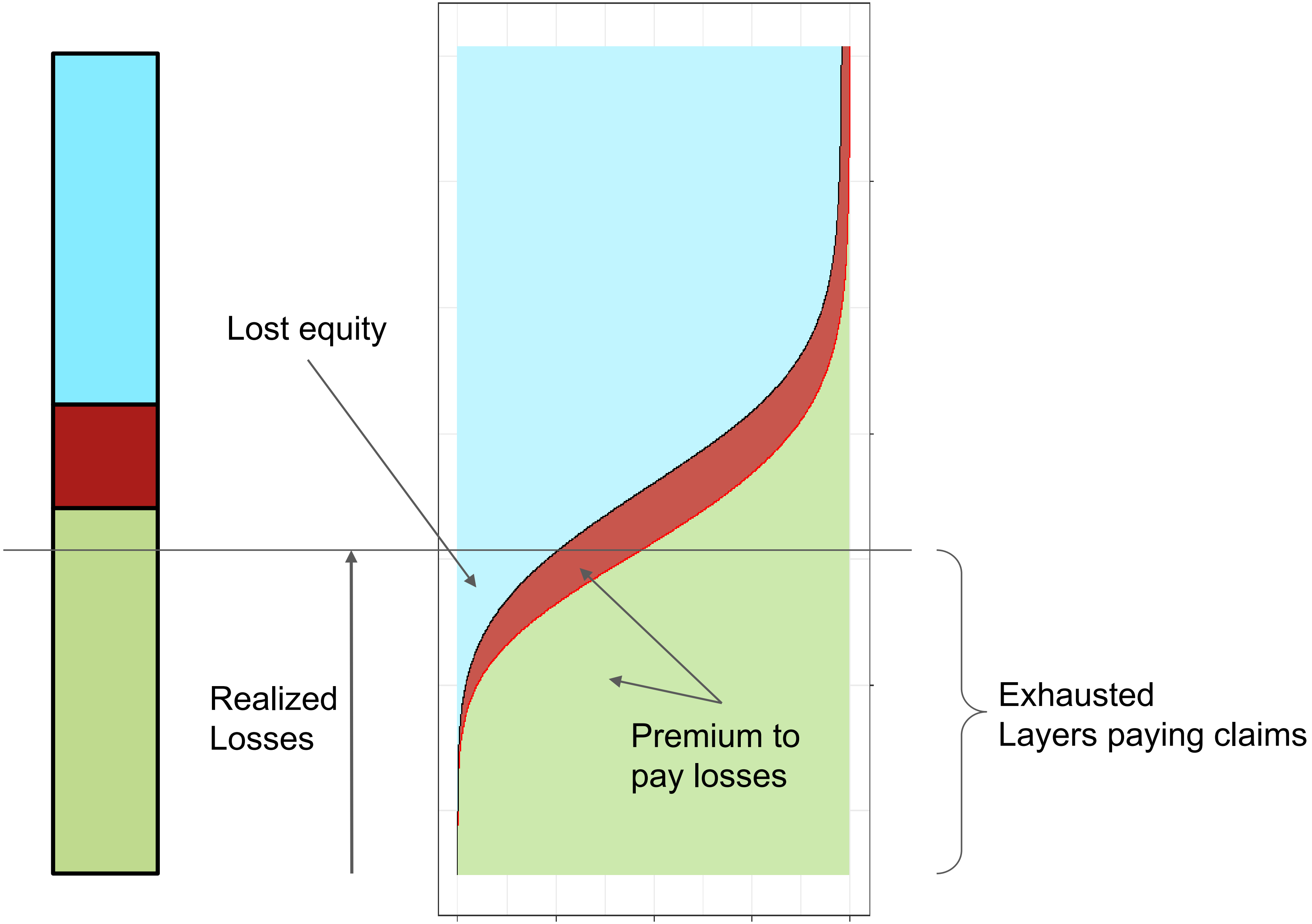




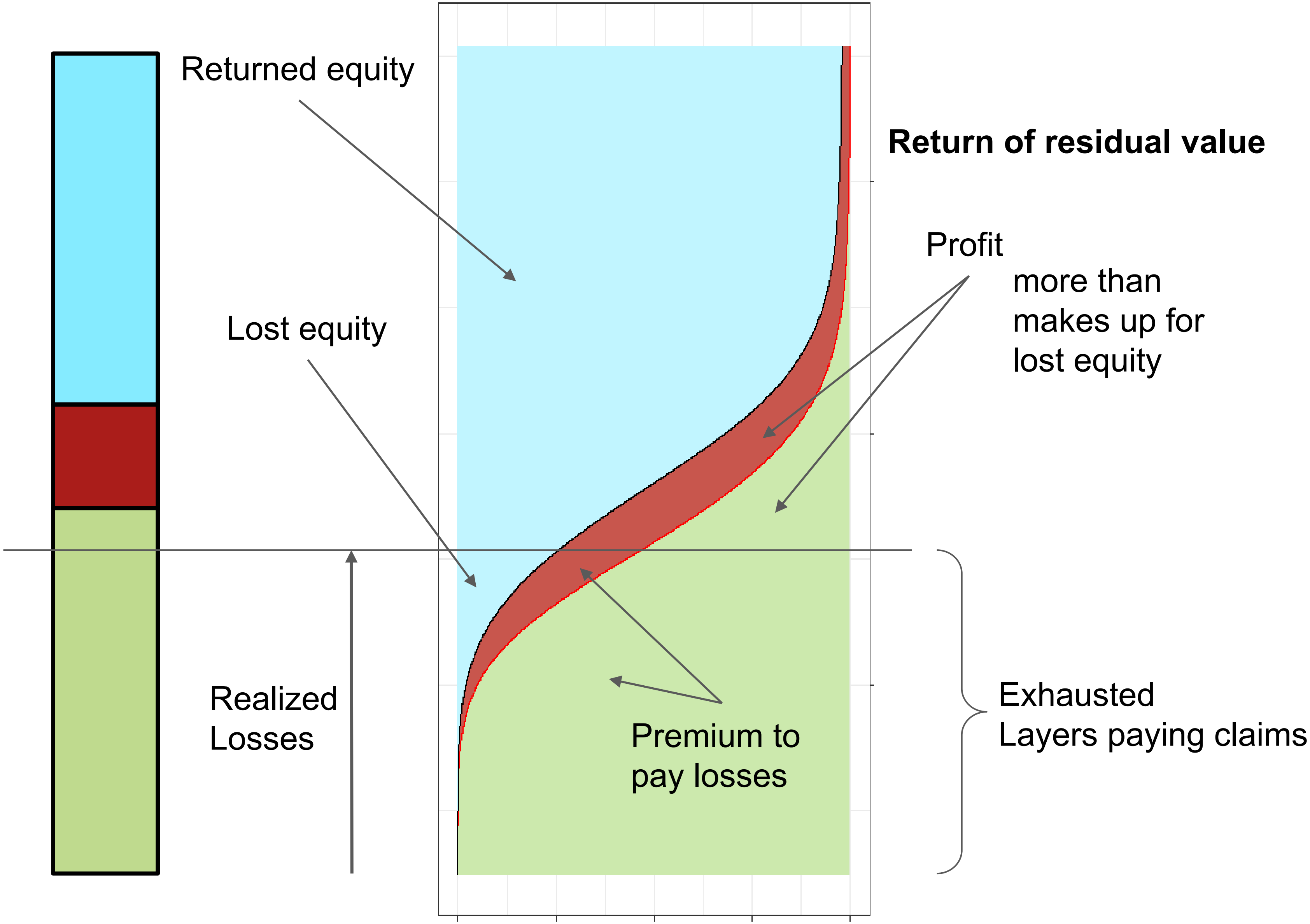
After operations financial reporting



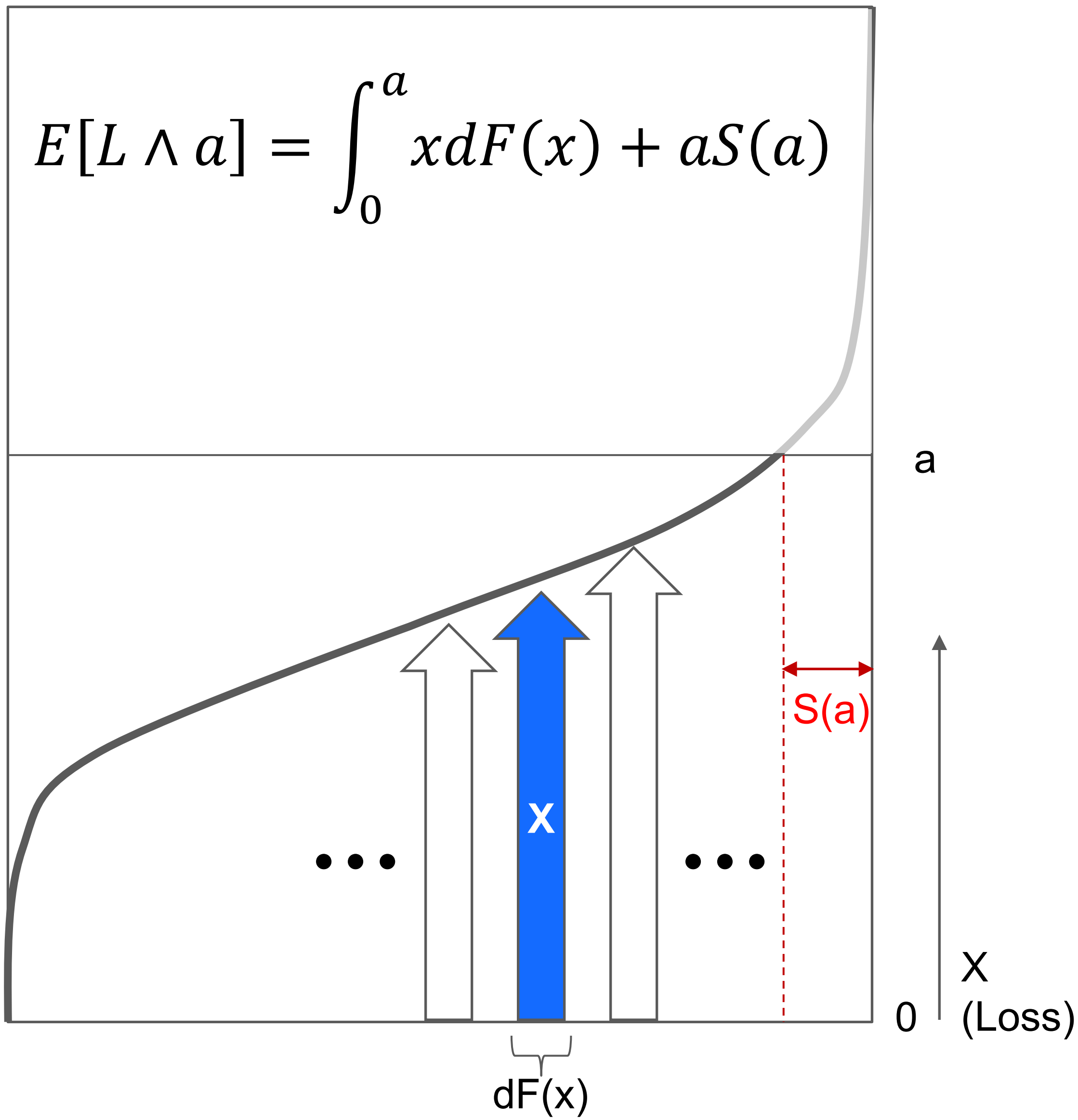
After operations financial reporting



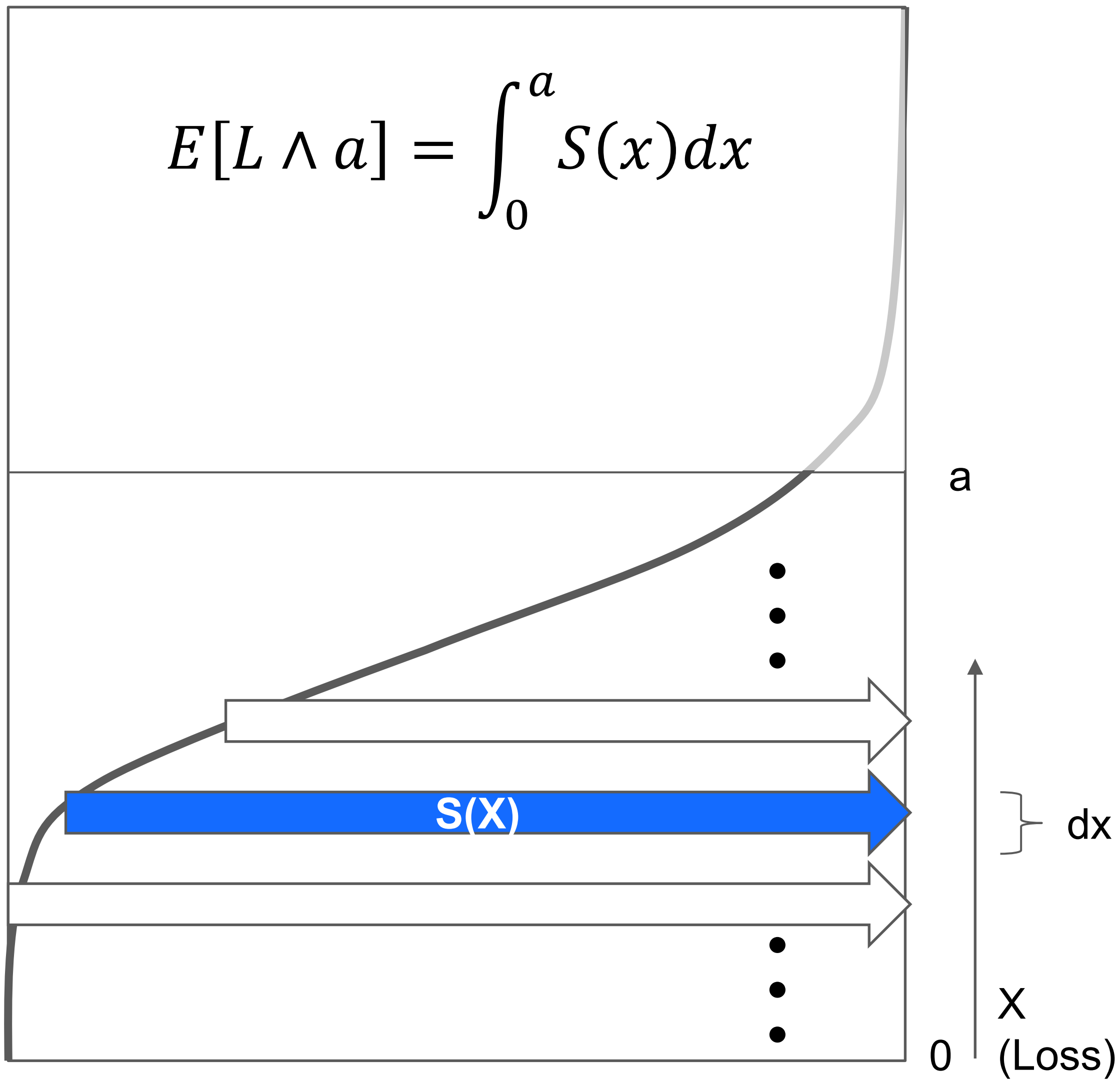
After operations financial reporting



Visualizing the expectation – conventional view



Visualizing the expectation – layer view



Probability distortion implies pricing

Expected loss (LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a x dF(x) + aS(a)$$

Probability distortion implies pricing

Expected loss (LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a x dF(x) + aS(a)$$

distorted probability

transformed cdf

Required premium
Distorted expected loss

$$E_g[L \wedge a] = \int_0^a g\big(S(x)\big)dx = \int_0^a x dG(x) + ag\big(S(a)\big)$$

Pause

- What you’ve seen so far:
  - Thinking about layers of assets
  - Each consists of premium + equity
  - Margin is cost of capital
  - Expected loss ***s*** determines layer funding
  - Functional relationship ***g*(*s*)**
  - … leads to Spectral Risk Measure => pricing

GUY CARPENTER

John.A.Major@guycarp.com

SPECTRAL RISK MEASURES (SRM) AND APPLICATIONS IN INSURANCE ERM – PART 2

Stephen J. Mildenhall

ASTIN Webinar March 25, 2020

Loss payments: who gets what in default?

- Sold insurance promises

$$X = X_1 + \cdots + X_n$$

- **Equal priority** payment to line *i* with assets *a*

$$X_i(a) = \begin{cases} X_i & X \le a \\ a\ (X_i/X) & X > a \end{cases}$$
$$= X_i \frac{X \wedge a}{X}$$
$$= \frac{X_i}{X} X \wedge a$$

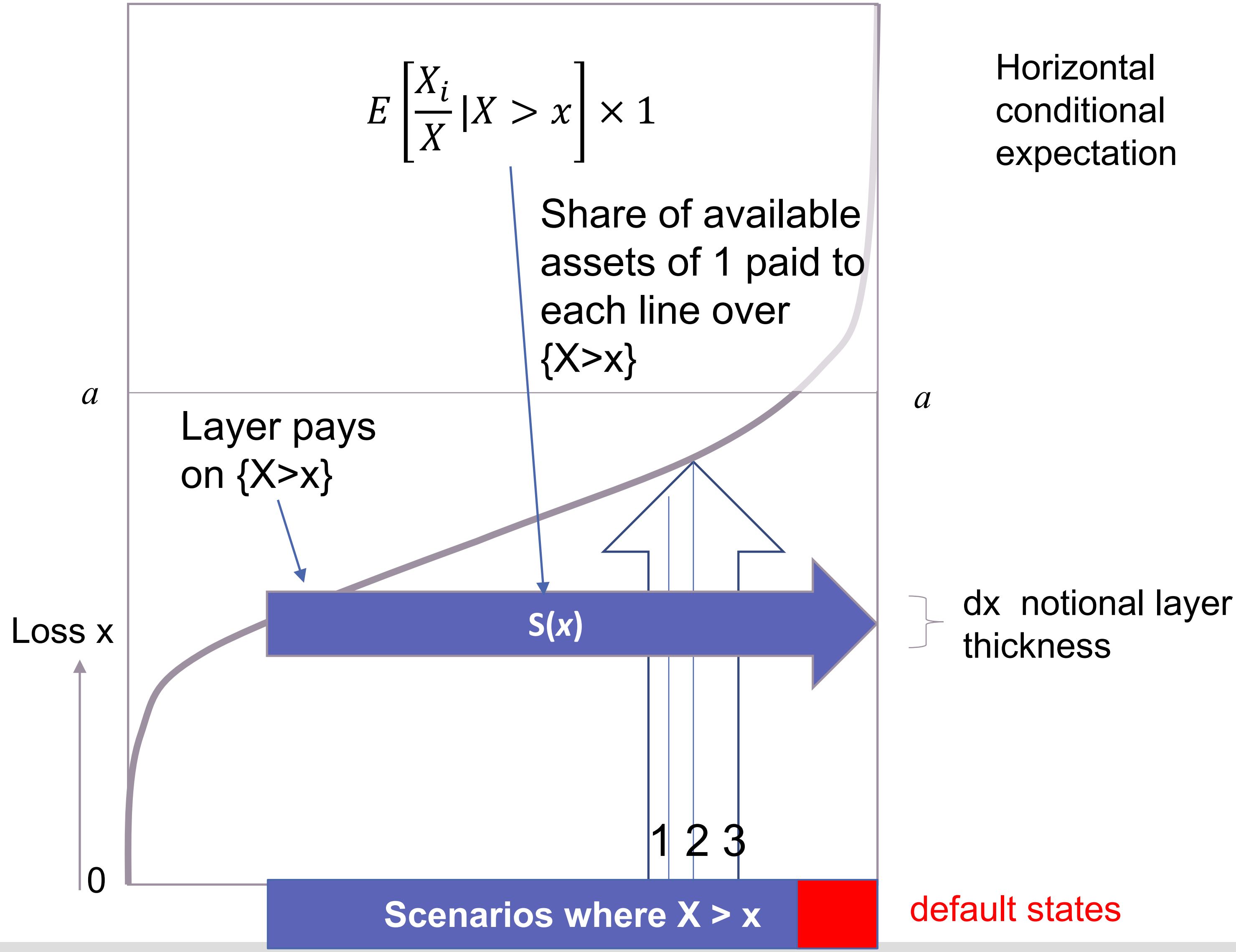
- $\dfrac{X \wedge a}{X}$ = fixed payment pro rata factor applied to loss by line
- $\dfrac{X_i}{X}$ = variable share of **available** assets
- $X \wedge a$ amount of assets **available** to pay claims
- $X_i(a)$ sum to $X \wedge a$, limited losses

Expected loss formulas

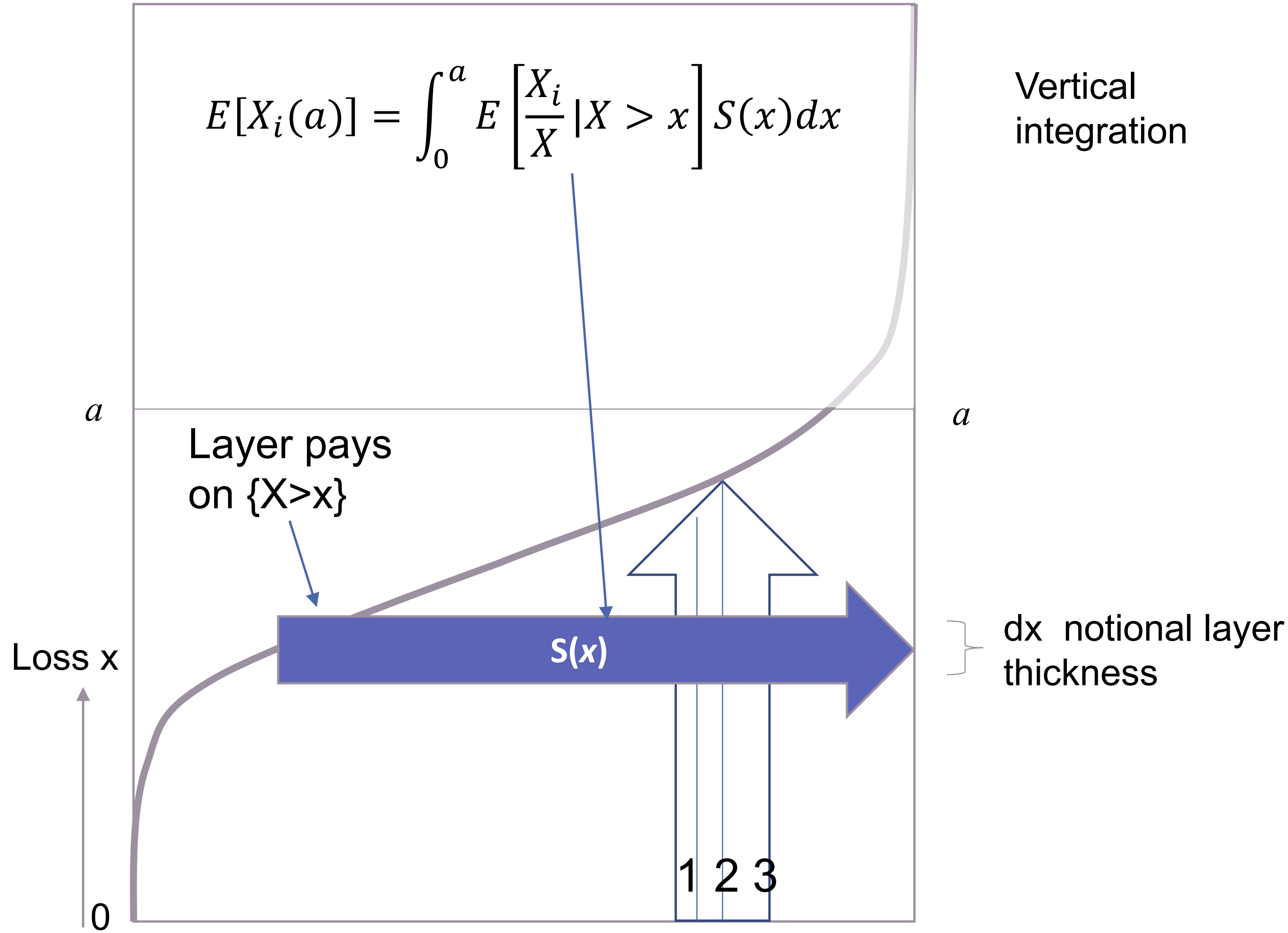
$$E[X \wedge a] = \int_0^a S(x)dx$$

$$E[X_i(a)] = ??$$

Visualizing expected loss payments by line and layer



Visualizing expected loss payments by line and layer



convex risk

Expected loss and premium by line and layer

$$\bar{L}_i(a) = E[X_i(a)] = \int_0^a \underbrace{E\left[\frac{X_i}{X} \mid X > x\right]}_{\alpha_i(x)} S(x)dx = \int_0^a \alpha_i(x)S(x)dx$$

$$\bar{P}_i(a) = E_g[X_i(a)] = \int_0^a \underbrace{E_g\left[\frac{X_i}{X} \mid X > x\right]}_{\beta_i(x)} g\big(S(x)\big)dx = \int_0^a \beta_i(x)g\big(S(x)\big)dx$$

α_i , β_i functions add-up: $\sum \alpha_i(x) = E\left[\frac{X_1+\cdots+X_n}{X} \mid X > x\right] = 1$

Loss, premium and margin by line, by layer and in total

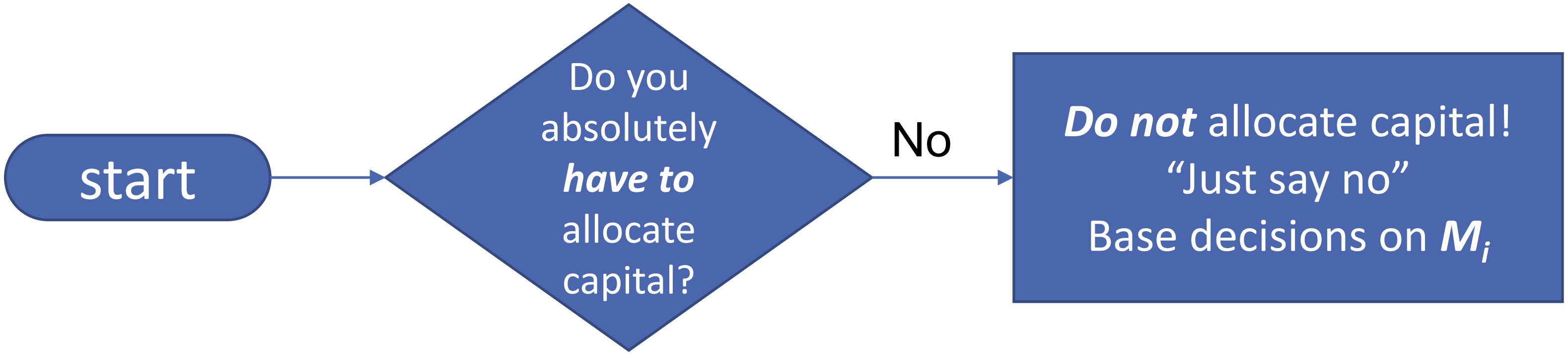
Loss Cost $$L_i(x) = \alpha_i(x)S(x)$$

Premium $$P_i(x) = \beta_i(x)g\big(S(x)\big)$$

$\Rightarrow$ Margin $$M_i(x) = P_i(x) - L_i(x)$$

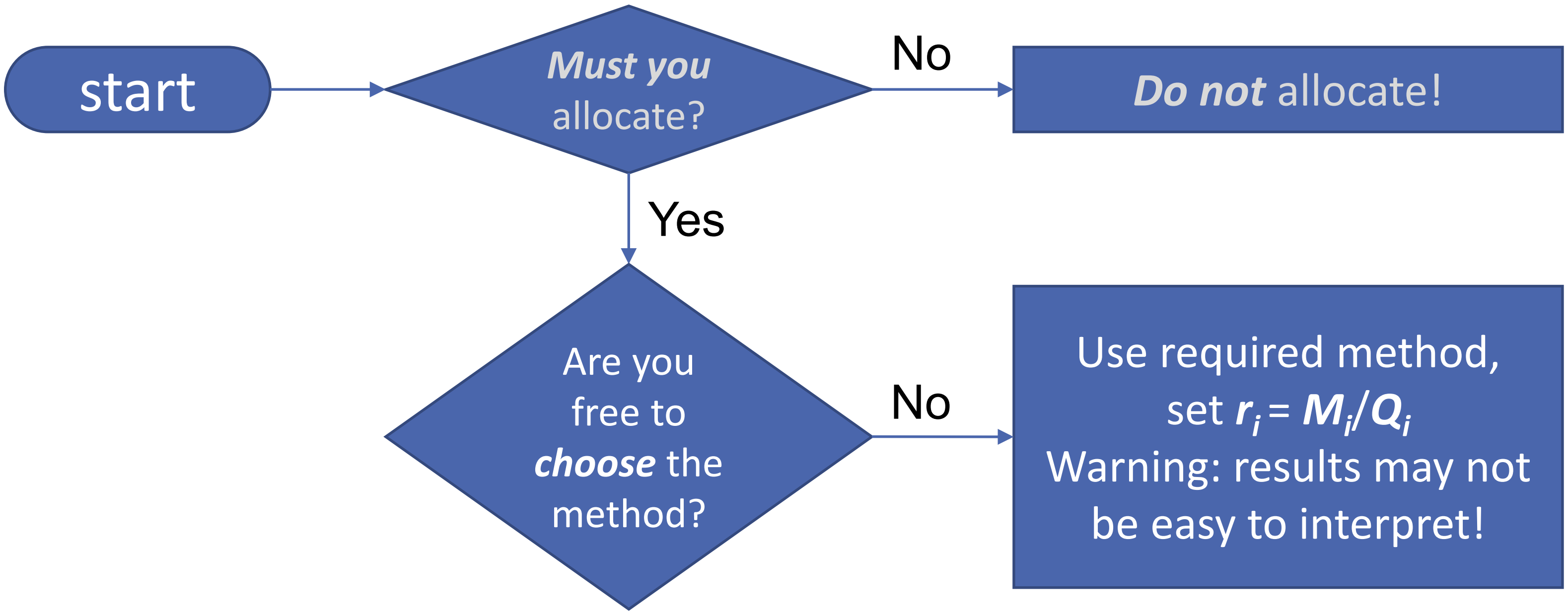
- **Integrate to get total expected loss, premium or margin as function of assets *a*: everything you need to price!**
- Assumptions
  - Price with *g*
  - Equal priority in default
- Independence of X_i **not** required
- All quantities add-up
- **No allocation...**

Capital allocation recommendation flowchart

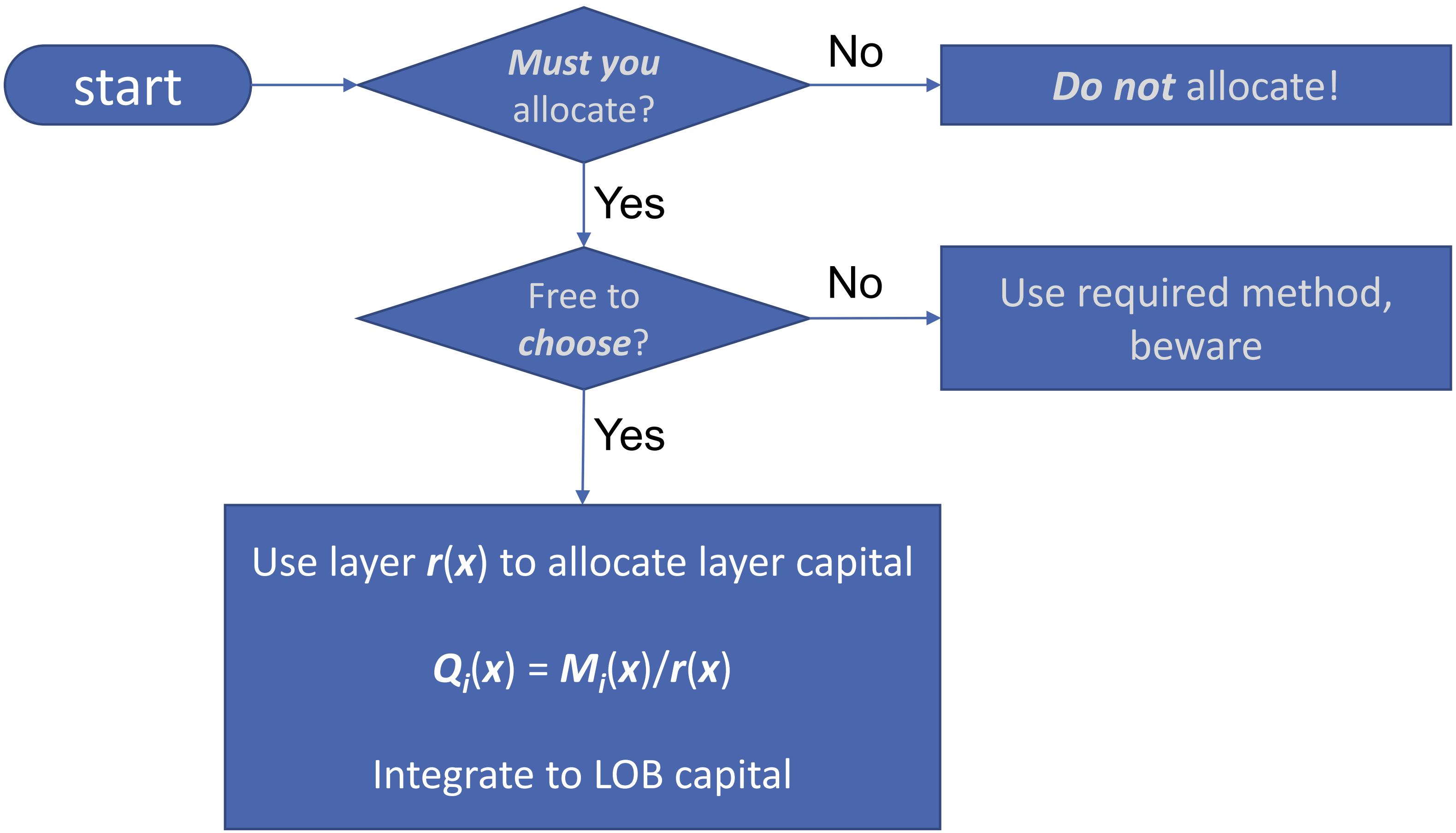


convex risk

Capital allocation recommendation flowchart



Capital allocation recommendation flowchart



convex risk

Law invariant assumption

A **law invariant** risk measure is function of the distribution; it does not distinguish by cause of loss within layer...therefore return can't vary by line within a layer

For a given **layer**, all LOBs must have the same ROE

$$r(x) = \frac{M(x)}{Q(x)} = r_i(x) = \frac{M_i(x)}{Q_i(x)}$$

Spectral risk measures are law invariant

convex risk

Implied layer capital allocation by line

$$r(x) = \frac{M_i(x)}{Q_i(x)} \Rightarrow Q_i(x) = \frac{M_i(x)}{r(x)} = M_i\ (x) / \frac{g(S(x)) - S(x)}{1 - g(S(x))}$$

$$= \frac{\beta_i(x) g\big(S(x)\big) - \alpha_i(x) S(x)}{g\big(S(x)\big) - S(x)} \Big(1 - g\big(S(x)\big)\Big)$$

Capital allocation — Capital in layer

Capital allocation can be negative!

- Margin can be negative if $\beta_i(x)$ sufficiently less than $\alpha_i(x)$

$$\frac{\beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)}{g\big(S(x)\big) - S(x)}$$

- When is $\beta_i(x) < \alpha_i(x)$? For relatively thin tailed lines!

Cost of capital varies by amount of assets

- Total cost of capital = total required margin is a function of total assets

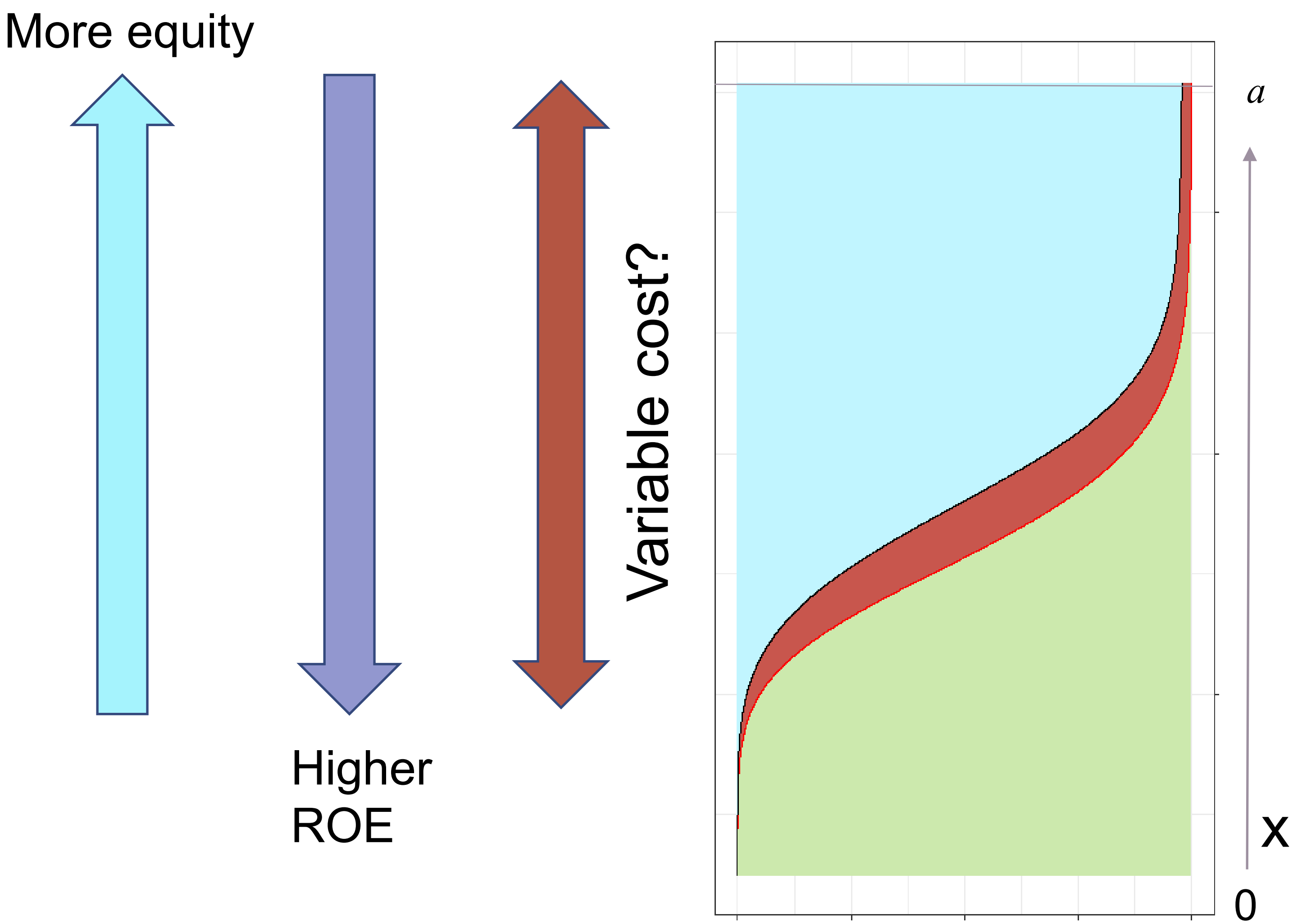
$$\bar{M}(a) = \int_0^{a} g\big(S(x)\big) - S(x)dx$$

- Total capital part of assets also varies

$$\bar{Q}(\mathrm{a}) = \int_0^{a} 1 - g\big(S(x)\big)dx$$

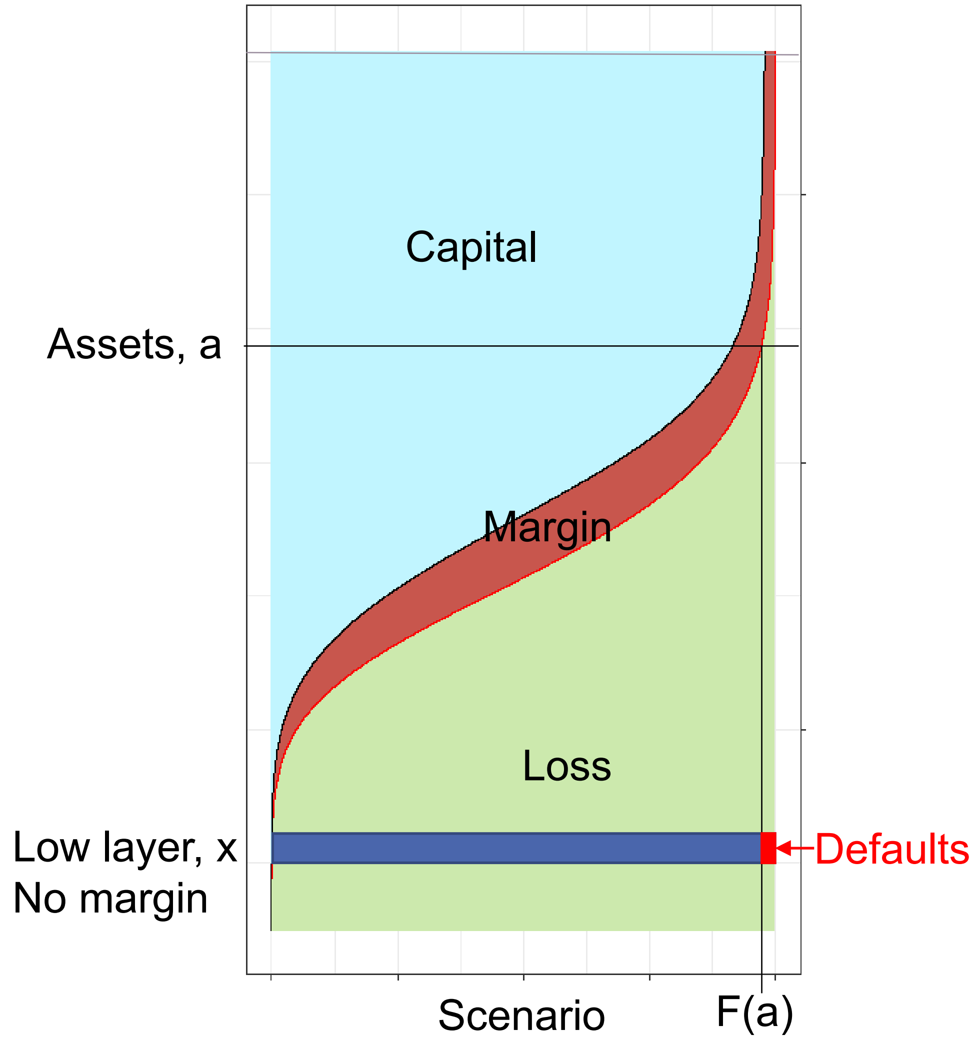
- Hence overall ROE also varies with total assets

Equity and margin vary by layer in complex manner



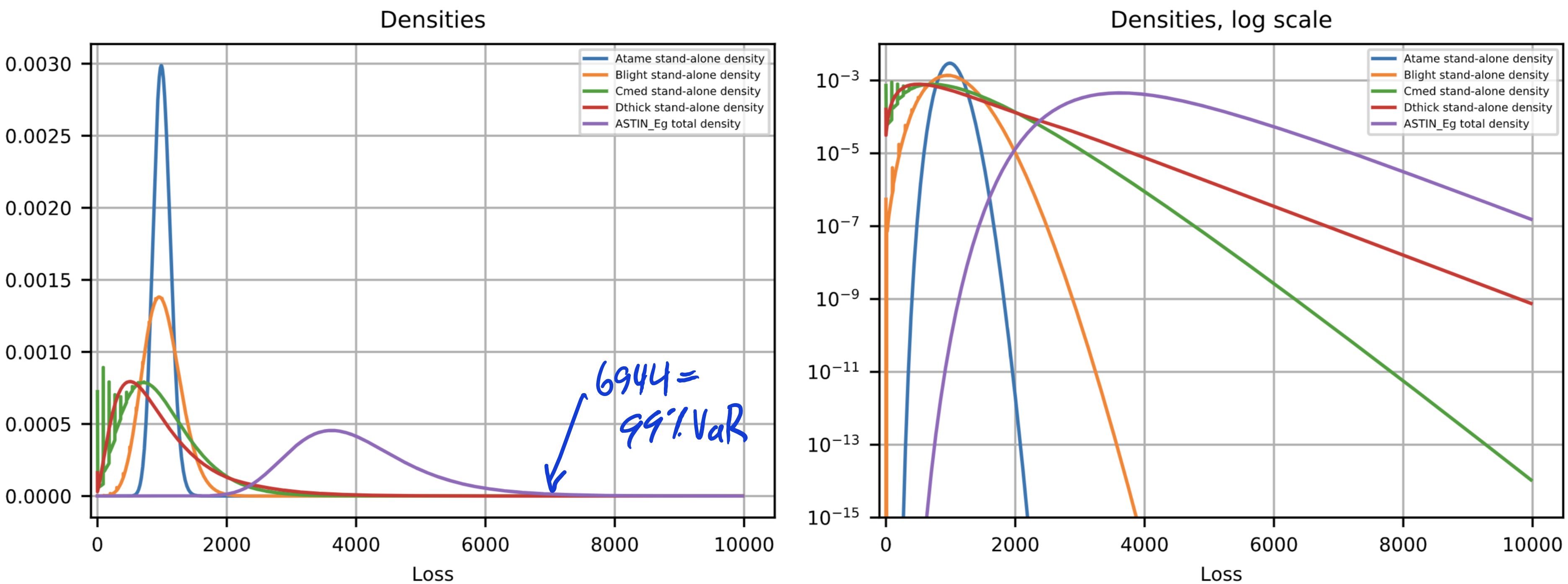
Two comments on loss layers

- Low asset layers have very small or zero total margin
  - Zero total margin does not imply zero margin by line
  - Sum of margin over lines = 0
- All layers respond in some default scenarios
  - Although layer is fully collateralized it does not pay at 100% on all scenarios



convex risk

Set up: four-line model



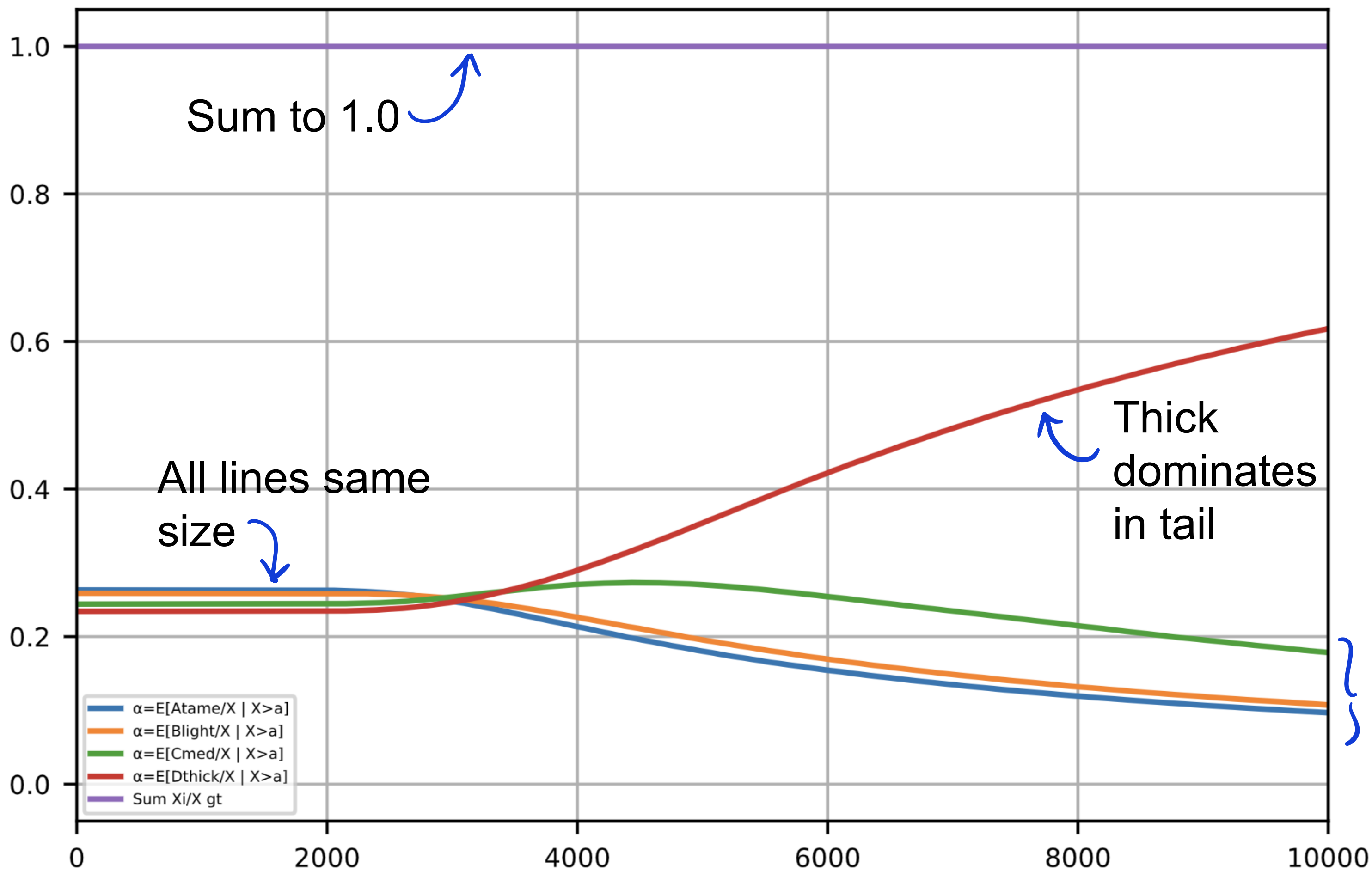
- Four independent lines: **Tame**, **Light**, **Medium**, **Thick**
- Each has expected (unlimited) loss 1000
- Tail thickness clear from right, log density, plot
- 99.0% VaR is 6,944, indicating a reasonable asset level
- Detailed statistics available in Appendix

Total Stats

Prem	4,261
Equity	2,683
Assets	6,944
Loss	3,993
LR	93.7%
ROE	10.0%

alpha function: calculates expected loss by line

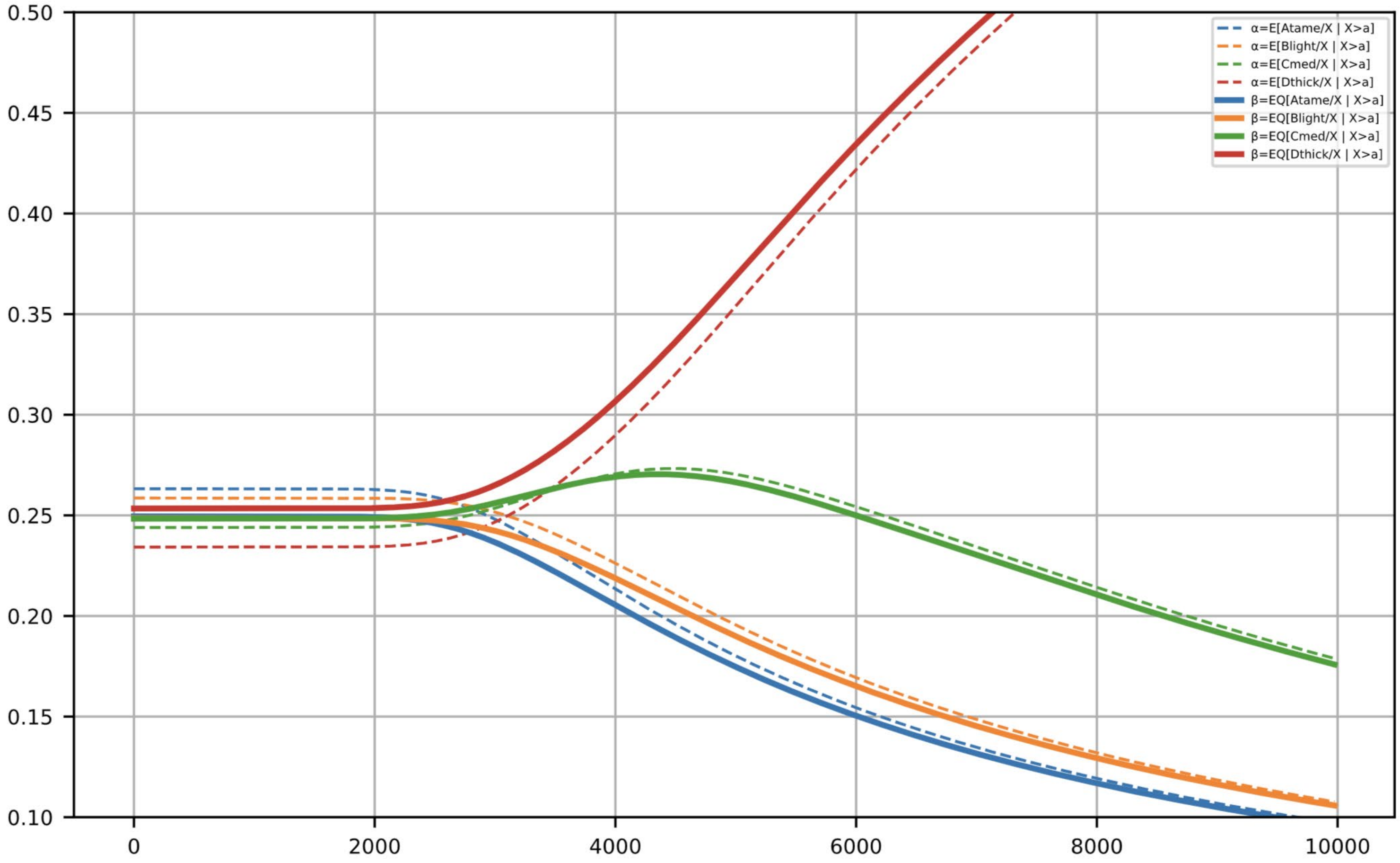
- $\alpha_i(x) = E[X_i/X \mid X > x]$ as a function of x asset level



Order of risks consistent with tail thickness

alpha, beta: g PH calibrated to 10% ROE at 99% VaR

- $\alpha_i(x) = E[X_i/X \mid X>x]$ (dashed line) as a function of assets *x*, calculates losses
- $\beta_i(x) = E_g[X_i/X \mid X>x]$ (solid line) calculates premium

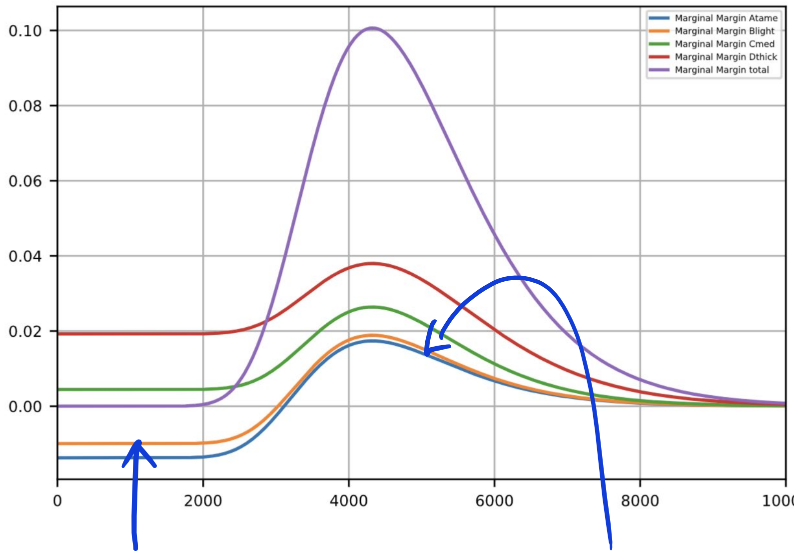


Negative margins possible when $\beta_i(x)<\alpha_i(x)$, tame, thin and medium above 3700

Margin by asset layer, by line

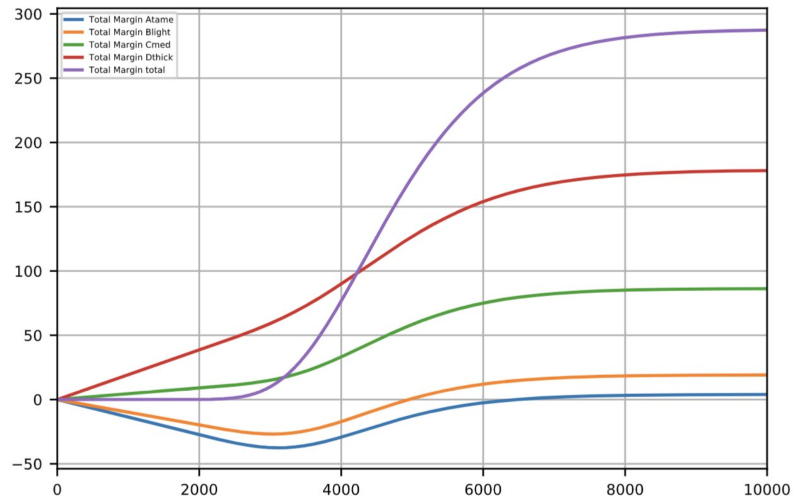
Layer Margin Density

$\beta_i(x)g(S(x)) - \alpha_i(x)S(x)$



Cumulative Margin

Integral of density



- Volatile lines gain by pooling with stable lines in low layers
- They pay a positive margin to compensate stable lines for worse coverage

- All lines benefit at high layers because higher equity to premium ratio
- All lines pay positive margin for incremental capital

- Above ~6500 all lines pay positive total margin
- Tame line's total cost reduced reflecting coverage impacts of pooling with volatile lines
- Tame indicated 0.6% cost of capital, light 4.7%, medium 11.5% and thick 12.6%; overall cost calibrated to 10.0%
- See appendix for details

Conclusions

- Premium based on **fair value to customers** of contractual cash flows and not **marginal cost to insurer**
  - Marginal cost view generally different, driven by regulatory capital standard
- ROE varies by layer, line and amount of capital in a complex manner!
- You don't need to allocate capital to price!
- Link to videos: http://go.guycarp.com/cas2018
- Fully executable Python workbook with example: http://bit.ly/2TJs5id
- See forthcoming book / paper for details!

convex risk

Appendix

convex risk

Example: audit statistics

- P = percentiles
- Emp = result of FFT discretization
- Err in last three rows report difference between empirical output and exactly specified inputs

- All examples produced using aggregate Python package https://github.com/mynl/aggregate
- pip install aggregate
- Aggregate portfolio specification:

```
agg Atame     94 claims  50 xs 0   sev lognorm 14.835 cv 3.0 fixed
agg Blight 14.42 claims 100 xs 0   sev 100.02 * expon + 10   poisson
agg Cmed    1000 loss    90 xs 10  sev gamma 10 cv 6.0       mixed gamma 0.5
agg Dthick  1000 loss  2000 xs 50  sev invgamma 20 cv 5.0    mixed ig 0.5
```

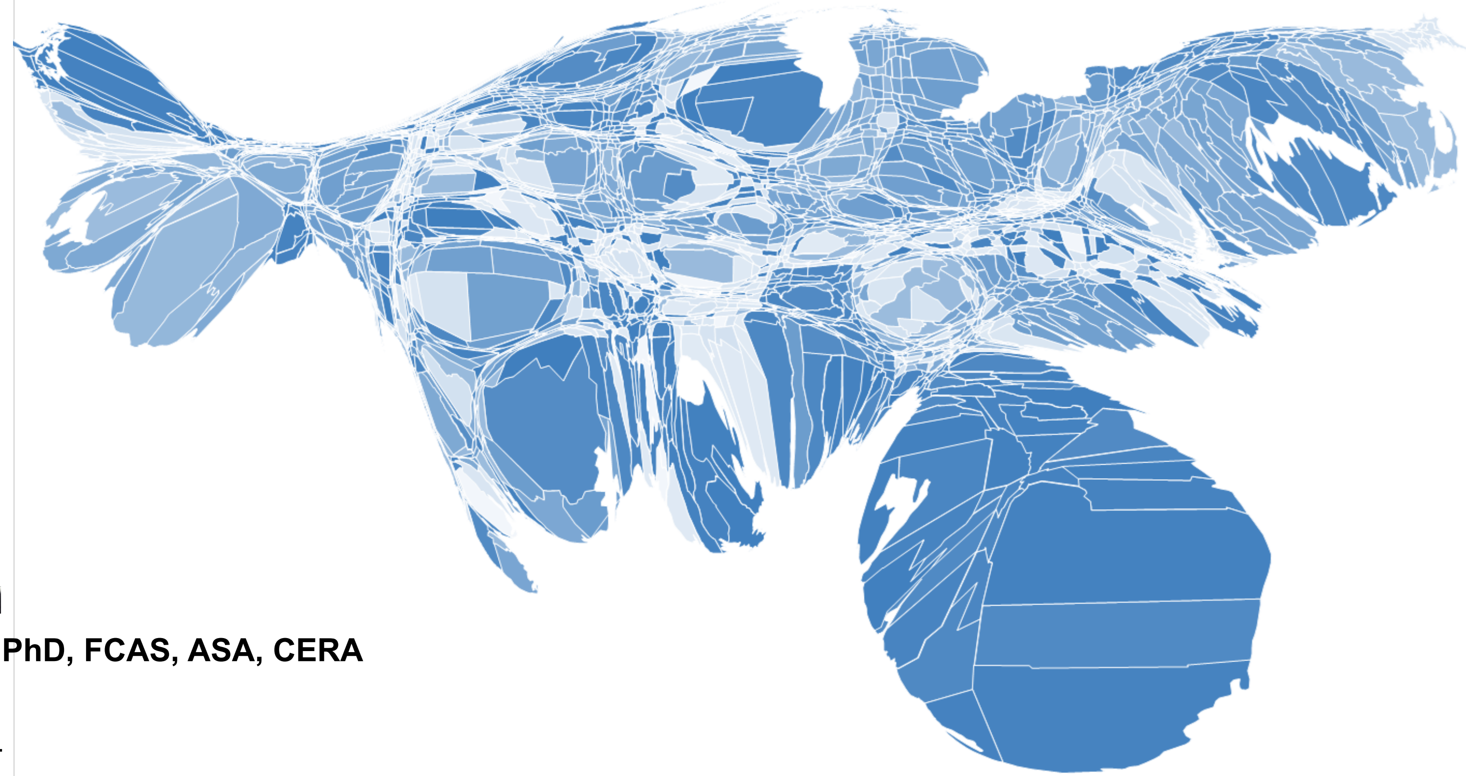
	Atame	Blight	Cmed	Dthick	total
Mean	1000	999.99	1000	1000	4000
CV	0.1335	0.29014	0.56943	0.7364	0.24604
Skew	0.18518	0.3144	1.0118	1.6902	0.91262
Limit	50	100	90	2000	2000
P99.9Est	1449	2034.1	3686.6	5270.1	8479.8
Sum probs	1	1	1	1	1
EmpMean	999.49	999.99	999.98	999.97	3999.4
EmpCV	0.13364	0.29015	0.56943	0.73641	0.24607
EmpSkew	0.18492	0.3144	1.0118	1.6902	0.91259
EmpEX1	999.49	999.99	999.98	999.97	3999.4
EmpEX2	1.0168e+06	1.0842e+06	1.3242e+06	1.5422e+06	1.6964e+07
EmpEX3	1.0524e+09	1.2602e+09	2.1594e+09	3.3016e+09	7.6463e+10
P90.0	1173	1381	1764	1963	5293
P95.0	1226	1502	2070	2464	5813
P99.0	1328	1739	2725	3542	6944
P99.6	1377	1856	3075	4141	7562
P99.9	1445	2019	3587	5044	8478
P99.99	1544	2260	4402	6532	9976
P99.9999	1712	2680	5956	9513	12958
MeanErr	-0.00051327	-3.5644e-06	-2.4109e-05	-2.7423e-05	-0.00014209
CVErr	0.00105	9.9876e-06	7.9081e-06	1.621e-05	0.00014084
SkewErr	-0.0014202	3.505e-06	3.8609e-08	3.9721e-06	-2.933e-05

Pricing results calibrated to 10% return @ 99% VaR

- Proportional hazard transform
- M = marginal
- T = total/cumulative
- L = loss
  LR = loss ratio
  M = margin = P - Q
  P = premium
  Q = capital
- Loss ratio net of expenses
- Capital and leverage ratios reasonable
- Substantial variation in ROE by line

MT	stat	Atame	Blight	Cmed	Dthick	total
M	L	0.0013563	0.0014958	0.0023543	0.0047907	0.0099971
M	LR	0.33879	0.33862	0.33698	0.32396	0.33108
M	M	0.0026471	0.0029217	0.004632	0.0099974	0.020198
M	P	0.0040034	0.0044175	0.0069863	0.014788	0.030195
M	PQ	0.031498	0.03149	0.031413	0.030807	0.031135
M	Q	0.1271	0.14028	0.2224	0.48002	0.9698
M	ROE	nan	nan	nan	nan	0.020827
M	alpha	0.13567	0.14963	0.23549	0.47921	nan
M	beta	0.13258	0.1463	0.23137	0.48975	nan
T	EPD	0.0008408	0.00092862	0.0014879	0.003434	0.0016729
T	L	998.65	999.06	998.49	996.54	3992.7
T	LR	0.99838	0.98368	0.92396	0.85579	0.93703
T	M	1.6253	16.571	82.17	167.93	268.3
T	P	1000.3	1015.6	1080.7	1164.5	4261
T	PQ	3.555	2.8579	1.5102	0.87512	1.5882
T	Q	281.38	355.38	715.57	1330.6	2683
T	ROE	0.0057763	0.046629	0.11483	0.1262	0.1

Contact Information



Stephen Mildenhall, PhD, FCAS, ASA, CERA
Convex Risk LLC
New York, NY 100024
+1.312.961.8781 cell
steve@convexrisk.com

Graphic note: County size scaled to AAL estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm